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AN ANALYSIS OF THE EFFECTIVENESS OF THREE MATHEMATICS PROGRAMS AT THE GRADE EIGHT LEVEL

by

Donald Bruce Harrison

A THESIS
SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "An Analysis of the Effectiveness of Three Mathematics Programs at the Grade Eight Level," submitted by Donald Bruce Harrison in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of the study was to gather information that could be used as a basis for evaluating the relative effectiveness of three mathematics programs, two "modern" and one "traditional," at the grade eight level.

In September, 1963, the *Cooperative School and College Ability Tests*, Level Four, and the *Iowa Tests of Basic Skills*, Form 1 (grade eight Arithmetic Concepts and Problem Solving sections) were administered to eleven classes of grade eight students studying *Seeing Through Mathematics*, to five classes of grade eight students studying *Exploring Modern Mathematics*, and to six classes of grade eight students studying *Winston Mathematics*. In March, 1964, the *Iowa Tests of Basic Skills* were readministered to the same students along with a *Special Mathematical Understandings* test that had been developed by the investigator. Two-way, unweighted means analyses of variance and covariance were carried out on the data to test the null hypotheses.

It was found that, as a group, the *Exploring Modern Mathematics* students scored significantly higher on the *Special Mathematical Understandings* test than the *Seeing Through Mathematics* students did, and the *Seeing Through Mathematics* students scored significantly higher on the

same test than the Winston Mathematics students did. The mean scores of all three groups of students on the Iowa Arithmetic Concepts test did not differ significantly. On the Iowa Problem Solving test, which is largely computationally oriented, the scores obtained by the Exploring Modern Mathematics students and by the Winston Mathematics students did not differ significantly. However, as a group, the Seeing Through Mathematics students scored significantly lower than the Winston Mathematics students on the Iowa Problem Solving test but not significantly lower than the Exploring Modern Mathematics students.

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D. B. H.

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CHAPTER I

THE NATURE AND SIGNIFICANCE OF THE PROBLEM

I. INTRODUCTION

Early in the 1950's, groups of mathematics teachers, mathematicians, and psychologists began to produce radically new elementary and secondary school mathematics curricula. Among the more prominent and influential of these groups have been: the University of Illinois Committee on School Mathematics, organized in 1951; the Ball State Experimental Program, initiated in 1954; the Commission on Mathematics of the College Entrance Examinations Board, established in 1955; the University of Maryland Mathematics Project, organized in 1957; and the School Mathematics Study Group, formed in 1958[1]. In the literature one finds both supporters for and critics of the proposals made for curricular reform in school mathematics by each of these groups. However, there is a consensus of opinion on basic ideas.

For instance, critics and supporters alike express dissatisfaction with the results of traditional teaching

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[1] Emmet D. Williams, "Comparative Study of SMSG and Traditional Mathematics" (unpublished Doctoral Dissertation, The University of Minnesota, Minneapolis, 1962), pp. 1-2.

of school mathematics which emphasizes rote-learned rules and computational skills, which fails to appreciate the unity in mathematics, and which does little to develop student enthusiasm. Traditional mathematics courses have been described as bags of unrelated tricks. Student attitudes towards mathematics in such courses have been found to be largely of a negative nature. There has been general dissatisfaction with the level of mathematical competence of high school graduates.

Another universally held view is that students must be given not only the basic mathematical skills but also a deeper understanding of the basic concepts and structure of mathematics. Furthermore, it is felt that mathematics can and should be made more attractive and interesting to students. In this era of phenomenal technological progress we cannot afford to turn students away from mathematics by subjecting them to courses that do not convey a spirit of adventure and discovery and that do not effectively prepare them to meet society's demands. As one curricular reform group asserts:

> The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; and understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty

> his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand of many of them.[2]

While there is substantial agreement as to the objectives of current school mathematics reform, there are conspicuous differences among the reform groups in their approach to and handling of mathematical ideas. Perhaps most noticeable of the differences among new junior high school mathematics programs is the degree to which symbolism and set notation is employed. Each program subscribes to the desirability of an intuitive, discovery-oriented approach to the development of concepts, but some programs seem to remain at the intuitive, non-symbolic level longer than others. Frequently mentioned in the literature is the warning that the trend toward increased rigour and formalism evidenced in some programs may kill mathematical intuition and interest, virtually sterilizing the subject. There is also some concern about whether or not increased time spent on the development of understanding with the attendant decreased drill time will result in poorer student facility with traditionally valued skills. Besides, one might ask whether the stated objectives of the

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[2] School Mathematics Study Group, Newsletter No. 4, March, 1960 (New Haven: Yale University, 1960), p.3.

new mathematics programs are actually being met. There is need for experimental evidence in these areas of concern, particularly because adaptations of the new programs are appearing in commercial textbook form and curriculum decision-making committees must be able to assess the effectiveness of the new texts in order to make reasonable choices.

Over the past few years, the Alberta Junior High School Mathematics Subcommittee has thoroughly investigated the problem of mathematics curriculum reform. In order to make known its views on the subject, the Subcommittee produced a bulletin in which it presented its "Guidelines for Revised Junior High School Mathematics Curriculum."[3] These "Guidelines," prepared in April, 1962, contain recommendations that the junior high school mathematics curriculum should include:

1. the concept of sets wherever possible,
2. a treatment of numeration systems designed to give the student an understanding of the base-ten numeration system and an appreciation of the history of number development,
3. an intuitive development of basic geometric relationships from the point-set approach,
4. study of the elements, operations and laws of operations in the natural, rational, integral and real number systems,
5. the development of problem solving ability with

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[3] Alberta Junior High School Mathematics Subcommittee, Junior High School Mathematics Bulletin, April, 1963, pp. 20-21. (Mimeographed.)

emphasis on a logical approach to problems, on the ratio or rate-pair approach, where applicable, on inequalities, and on graphical solutions,

6. treatment of measurement and the measurement process,
7. at least a brief treatment of the strategies of logic used in proving statements,
8. a brief intuitive treatment of one or more of: permutations, combinations, probability, statistics, and series, and
9. conventional topics such as reviews of previously learned concepts, application of per cent to interest, merchandising, etc., and statistical graphs.

The Subcommittee has singled out two new commercial textbook series that more or less fulfill the expectations for a junior high school mathematics curriculum expressed in the "Guidelines." Out of this situation grew the need for the study outlined on the succeeding pages.

II. THE PROBLEM

Do students using the new commercial textbooks develop more understanding of the mathematical ideas underlying the course content than those students studying the conventional text? How do students studying the new commercial textbook materials compare with students studying the traditional, currently authorized textbook materials with respect to achievement as measured by a conventional mathematics test? Do groups of students studying the two new commercial textbooks differ from one another with respect to achievement on a conventional mathematics test

or with respect to demonstrable mathematical understandings? The investigation presently being reported was an attempt to gather objective evidence that might shed some light on the answers to these questions.

Delimitation of the Problem

Since classes and teachers that were using the new mathematics text materials at the grade eight level were readily accessible during the 1963-64 school term, the study was particularly concerned with that grade level in pursuing answers to the questions posed above. The new commercial text materials involved in the study were Books 1 and 2 of Seeing Through Mathematics[4] (referred to hereinafter as STM), Books 1 and 2 of Exploring Modern Mathematics[5] (referred to hereinafter as EMM), and Winston Mathematics, Intermediate 1 and Intermediate 2[6] (referred to hereinafter as CON for conventional text or control group).

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[4] Henry Van Engen and others, Seeing Through Mathematics, Book 1 (Canadian edition; Toronto: W. J. Gage Limited, [n.d.], 2 parts; and Henry Van Engen and others, Seeing Through Mathematics, Book 2 (Chicago: Scott, Foresman and Company, 1962), 2 parts.

[5] Mervin L. Keedy, Richard E. Jameson, and Patricia L. Johnson, Exploring Modern Mathematics, Book 1 (New York: Holt, Rinehart and Winston, Inc., 1963); and Mervin L. Keedy, Richard E. Jameson, and Patricia L. Johnson, Exploring Modern Mathematics, Book 2 (New York: Holt, Rinehart and Winston, Inc., 1963).

[6] H. L. Stein and others, Winston Mathematics, Inter-

Null Hypotheses

The major null hypotheses tested were:

1. On a special mathematical understandings test, there are no significant differences
    a) among the group mean scores obtained by STM, EMM, and CON students,
    b) among scholastic ability level mean scores.
2. On a conventional, standardized test of arithmetic concepts, there are no significant differences
    a) among the group mean scores obtained by STM, EMM, and CON students,
    b) among scholastic ability level mean scores.
3. On a conventional, standardized test of arithmetic problem solving, there are no significant differences
    a) among the group mean scores obtained by STM, EMM, and CON students,
    b) among scholastic ability level mean scores.

III. SIGNIFICANCE OF THE PROBLEM

Nelson struck at the heart of the problem when he wrote ". . . it is one thing to proclaim the importance of understanding and quite another to produce it in the classroom."[7] The authors of new text materials may have set out to achieve the most acceptable goals in teaching for understanding, but it is merely a matter of conjecture whether or

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mediate 1 (Toronto: The John C. Winston Company, Limited, 1954, 1957); and H. L. Stein and others, Winston Mathematics, Intermediate 2 (Toronto: The John C. Winston Company, Limited, 1954, 1957).

[7] L. Doyal Nelson, "Relation of Textbook Difficulty to Mathematics Achievement in Junior High School" (unpublished Doctoral Dissertation, The University of Minnesota, Minneapolis, 1962), p. 3.

not they have achieved some measure of success until supporting experimental evidence is made available. We are fairly certain that the use of conventional (traditional) text materials does not generally lead to the development of desired student mathematical understandings, but is there experimental evidence to support this feeling? Do the new text materials help teachers to lead students toward any better or any more mathematical understandings? Since these are questions which must be considered before school authorities will be willing to accept the new materials on a very large scale and since the incorporation of better teaching materials into the mathematics curriculum is probably one of the most pressing needs in school mathematics, a major portion of the study presently being reported was devoted to attempts to provide some answers to these questions.

There is also natural concern about whether the decreased emphasis on "social arithmetic" and computational drill in the new programs will result in any significant losses in a student's ability to use his mathematics in social problems or in accurate computation. A portion of the study was devoted to consideration of these factors.

There is evidence in the literature of increasing awareness that experimental studies are needed to supplement the usual bases for making curricular decisions (e.g., text-

book examination, teacher opinion, and conjecture). Realizing that the drive to improve school mathematics is progressing rapidly and that the pressure on groups that must make decisions regarding the new programs available is becoming more intense, the need for experimental studies in this area can hardly be overstated.

IV. EXPERIMENTAL SETTING

In the 1962-63 school term, the Alberta Junior High School Mathematics Subcommittee arranged to have a number of teachers in the province use STM, Book 1 and EMM, Book 1 with their grade seven classes. Several classes using the conventional text, Winston Mathematics, Intermediate 1, were chosen to function as a control group. In early October, 1962, and again in May, 1963, the Cooperative School and College Ability Test (SCAT), Level Four, and the grade seven mathematics section of the Sequential Tests of Educational Progress (STEP) were administered to all students involved in the experiment. Besides, a test constructed by the Subcommittee and designed to test student understanding of STM and EMM course content was administered to the students in all three instructional groups in May, 1963. The results of that study are summarized in Chapter II of this report.

The majority of the students involved in the 1962-63

grade seven experiment were also involved in the 1963-64 grade eight experiment. However, it was not possible to combine the two studies because the data for individual students collected in the 1962-63 study were no longer available when the 1963-64 study commenced. Furthermore, because school timetables could not be arranged so that two of the control classes could remain intact for another term, these classes were replaced before the September, 1963 testing. Also, two STM classes were added to the experiment in September. Since the Subcommittee was dissatisfied with the characteristics of forms A and B of the STEP test, arithmetic sections from the Iowa Tests of Basic Skills were chosen for use in the 1963-64 study in place of the STEP test. Considering these factors, it was decided to treat the 1963-64 study as being independent from the 1962-63 study.

Twenty-two classes of grade eight mathematics students, from various Alberta towns and cities, participated in the 1963-64 experiment. Eleven of these classes studied parts of STM, Books 1 and 2, and five classes studied parts of EMM, Books 1 and 2. The remaining six classes studied the currently authorized textbook, Winston Mathematics, Intermediate 2.

At the beginning of the experiment, in late September, the Cooperative School and College Ability Test (SCAT),

Level Four, and the grade eight Arithmetic Concepts and and Problem Solving sections of the Iowa Tests of Basic Skills were administered to all participating pupils. In early March, 1964, the Iowa Tests of Basic Skills were re-administered to all participating pupils. Furthermore, a special test, prepared by the investigator and designed to measure the types of mathematical understandings currently considered desirable at the grade eight level, was administered to all students in early March.

The findings of the experiment are based upon statistical analyses of the data gathered from the testing.

V. DEFINITION OF TERMS

In general, the meaning of each of the following terms and abbreviated forms of reference is indicated where it is first used in the report, but, for easy reference, the following list is presented at this point.

_AC_ refers to the "Arithmetic Concepts" section of the _Iowa Tests of Basic Skills_, grade eight level.

_AV_ refers to the average ability group as defined below for the purposes of this experiment.

Average ability group (or level) refers to those students involved in the experiment who had a SCAT Total score of 82 or less.

CON refers to the group of students in the experi-

ment who studied *Winston Mathematics* text materials.

COOP refers to the *Cooperative* (Mathematics) *Tests*.

DAT refers to the *Differential Aptitude Tests*.

EMM refers to the *Exploring Modern Mathematics* texts or the group of students studying them.

ETS is used in reference to the Educational Testing Service.

HAV refers to the high-average ability group defined below.

HI refers to the high ability group defined below.

High ability group refers to those students in the experiment who obtained SCAT Total scores of 96 or above.

High-average ability group refers to those students who obtained SCAT Total scores greater than 82 and less than 96.

IOWA refers to the *Iowa Tests of Basic Skills*.

"*Modern*" refers to school mathematics materials that convey the spirit and recommendations of contemporary mathematics curricular reform groups.

New school mathematics is used interchangeably with "modern" school mathematics.

PS refers to the "Problem Solving" section of the *Iowa Tests of Basic Skills*.

Q refers to the quantitative ability subtest of the SCAT test.

SCAT refers to the *Cooperative School and College Ability Test*.

SMSG refers to the School Mathematics Study Group.

SMU refers to the *Special Mathematical Understandings* test devised by the investigator.

STEP refers to the *Sequential Tests of Educational Progress*.

STM refers to the *Seeing Through Mathematics* text materials or the group of students studying them.

Traditional school mathematics is used to refer to the approach to the teaching of school mathematics that has not emphasized the structure of the subject and has relied largely on the teaching of rules.

UMMaP refers to the University of Maryland Mathematics Project.

V refers to the verbal ability subtest of the SCAT test.

WRA refers to the *Wide Range Achievement Test*.

VI. OUTLINE OF THE REPORT

The present chapter is an introduction to and preview of the study. Chapter II consists of a review of the literature that has a bearing on the study being reported. Chapter III includes a detailed description of the design of the experiment and the statistical procedures

used in analyzing the data. The results from the statistical analyses are presented in Chapter IV. Chapter V is devoted to a summary of the investigation along with conclusions, limitations, and implications for further research.

CHAPTER II

REVIEW OF THE LITERATURE

I. INTRODUCTION

In the past decade there has developed what has been called a "revolution" in school mathematics. To convey some idea of the nature, objectives, criticisms, and present state of this revolution in mathematics education, the first part of the present chapter is devoted to a review of views expressed by writers in the field of mathematics education from January, 1959 to the present. It is in such a climate that the present deliberations on curricular change in the province of Alberta must proceed.

The second part of the chapter is devoted to a review of research studies considered particularly relevant to the investigation presently being reported.

The sources of information for the reviews in this chapter were: (1) the Education Index (1959-1964), (2) mathematics education periodicals and newsletters (1959-1964), (3) published reviews of research in mathematics (1954-1964), and (4) unpublished doctoral dissertations.

II. REVIEW OF CURRENT VIEWS ON SCHOOL MATHEMATICS

The Case for Reform

The Commission on Mathematics of the United States College Entrance Examination Board has attempted to summarize current views on school mathematics as traditionally taught.

> There is abundant evidence of public dissatisfaction with mathematics programs, curricula, and instruction. Many writers have claimed that the general quality of mathematics teaching is in need of improvement, and that the level of mathematical competence of American high school graduates is low. Colleges complain that entering freshmen are poorly prepared in mathematics. The requirements in mathematics for high school graduation in many states are exceedingly limited. It is alleged that too small a percentage of high school students is enrolled in mathematics courses, and that many students dislike mathematics. Complaints such as these became very widespread after the launching of the first earth satellite.[1]

A college mathematician has said that mathematics, commonly presented without motivation as a series of techniques, processes, and theorems, is isolated from all other bodies of knowledge and human interest so that it seems of little value except as preparation to proceed through more unattractive material.[2] Begle has remarked that, in the present secondary school mathematics curric-

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[1] Commission on Mathematics, Program for College Preparatory Mathematics (New York: College Entrance Examination Board, 1959), pp. 6-7.

[2] Morris Kline, "Reform in the Mathematics Curriculum," Education Digest, 24:36, February, 1959.

ulum, students are taught skills by rote and no attempt is made to relate the skills as a unit.[3] According to Beberman, students taught efficient methods for manipulating symbols, without being given sufficient experience to become aware of the abstractions denoted by the symbols, perform well on tests which require them to demonstrate the skills in familiar settings, but the same students are at a loss when the familiar skill is hidden in a new situation or when some time has elapsed since the skill was learned.[4] In a research study cited by Brown and Kinsella, it was found that, for grade twelve students, "getting an acceptable answer was the primary concern; understanding the problem and its solution was secondary. Thus, there was a tendency to grasp at superficial clues and to employ learned methods of solution hastily and uncritically."[5] Another reviewer has cited several studies that tended to support the claim that students hold predominantly negative attitudes towards mathematics as it has been traditionally

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[3] "Revolution in Math," Overview, 1:57, July, 1960.

[4] Max Beberman, "Improving High School Mathematics Teaching," Educational Leadership, 17:164, December, 1959.

[5] Kenneth E. Brown and John J. Kinsella, Analysis of Research in the Teaching of Mathematics, 1957 and 1958, United States Department of Health, Education and Welfare, Bulletin 1960, No. 8 (Washington: United States Government Printing Office, 1960), p. 33.

taught in the schools.[6] It could be inferred from the preceding observations that, in general, traditional school mathematics programs have not been very successful.

With specific reference to the grade seven and eight levels, one writer stated that "the usual curriculum for these grades, it is felt, is boring to the good student and frustrating to the weak one. The usual subject matter in these grades is poor preparation for later courses."[7] In a review of research studies it was reported that over three-quarters of nearly 400 seventh and eighth grade pupils in a large city scored less than 50 per cent on a test of arithmetic meanings.[8] It would appear that the junior high school situation is no better than that in high school.

The case for reform seems strong enough merely on the basis that traditional programs have apparently been unable to achieve their own limited objectives, but one

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[6] Rosalind L. Feierabend, "Review of Research on Psychological Problems in Mathematics Education," _Research Problems in Mathematics Education_, United States Department of Health, Education and Welfare, OE-12008; Cooperative Research, Monograph No. 3 (Washington: United States Government Printing Office, 1960), p. 19.

[7] Edward G. Begle, "The School Mathematics Study Group," _National Association of Secondary School Principals Bulletin_, 43:28, May, 1959.

[8] Brown and Kinsella, _op_. _cit_., p. 12.

must also take into account the pressure of the advance of human knowledge. "The subject matter of mathematics is constantly growing," wrote Brown, in 1954, "not only in the area of advanced mathematics, but also in the area of elementary mathematics."[9] Others have warned that society is in the midst of a technological upsurge and many new applications of mathematics to practical uses are being found. There is a need to articulate changes down from the universities into the high school, junior high school and elementary school.[10]

If it is accepted that the traditional manner of teaching mathematics has been and will continue to be inadequate, then ways of teaching the subject must be found that are more effective. This leads to a consideration of new or "modern" mathematics programs.

New or "Modern" Mathematics Programs

The term "modern" mathematics is widely used, but, since it has caused misunderstandings, it is avoided by many writers. Some writers have tried to outline what the

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[9] Kenneth E. Brown, "Improved Programs in Mathematics Require Inservice Education for Teachers," The Mathematics Teacher, 54:86, February, 1961.

[10] Dan T. Dawson, and William F. McClintock, "The Revolution in Mathematics," National Elementary Principal, 43:15-16, September, 1963.

term "modern" mathematics implies when used in reference to school mathematics.

One description of "modern" mathematics programs follows:

> There is concern with equalities, inequalities, set vocabulary, basic laws, properties of number, factors and primes, systems of enumeration, number line, Venn diagrams, etc. The stress on precision in vicabulary [*sic*] seems to run through each of these programs or experiments. Discovery is emphasized to lead the pupil to a more thorough understanding. If he finds out for himself the why's of mathematics he certainly will have a better command of the subject and will be more likely to arrive at conclusions as a result of analysis and reason. All of these factors seem to have a bearing on any program that calls itself modern mathematics.[11]

The head of a state teacher's college describes modern mathematics as: a careful reorganization of the subject matter of school mathematics into a logical, integrated sequence; the selection of appropriate language for precision and clarity of expression; more appropriate placement of meaningful drill on the basis of new knowledge about how pupils learn; a "point of view, an *approach* to the teaching of mathematics as an integrated structure"; a striving by imaginative teachers to teach for pupil discovery.[12]

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[11] L. B. Fisher, "How Curriculum Builders View New Mathematics Ideas," *School Science and Mathematics*, 64:33, January, 1964.

[12] Ross A. Nielsen, "Evaluating the Secondary Mathematics Program," *The North Central Association Quarterly*, 38:249-250, Winter, 1964.

According to one writer, modern mathematics does not mean abandonment of all or even much of what has been previously taught nor does it necessarily mean introduction of radically new mathematics materials.[13]

So far, the spirit of "modern" school mathematics, in general, is fairly apparent, but how well do specific curricular recommendations reflect this spirit?

The Nature of Current Curricular Recommendations

Experimental Programs for the Seventh and Eighth Grades. The authors of the Ball State Experimental Program, begun in 1954, have stated that their primary objective is "to lay a foundation for the continuing study of mathematics," not to teach cheque writing, meter-reading or the making and interpreting of bar and circle graphs.[14] The topics treated in the eighth grade course include: the role of symbolism in mathematics, student discovery of the fundamental laws of arithmetic, a little elementary number theory with informal proofs, intuitive treatment of ratio and proportion situations, rational numbers, irrational numbers as infinite decimals, set concepts that are found useful

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[13] Phillip S. Jones, "The Mathematics Teacher's Dilemma," Education Digest, 24:37, May, 1959.

[14] Charles Brumfiel, Robert Eicholz, and Merrill Shanks, "The Ball State Experimental Program," The Mathematics Teacher, 53:76, February, 1960.

in the discussion of the solution of equations and inequalities, sentences in one and two variables, sets of ordered pairs, negative numbers, signed rational numbers, applications and word problems.[15] According to the authors, the material is _concept-centered_ rather than _symbol-centered_.[16]

The University of Maryland Mathematics Project (UMMaP), begun in the fall of 1957, has produced text materials for seventh and eighth grade mathematics. The directors have reported that the concept of a mathematical system is at the core of UMMaP courses. Students are given opportunities "to recognize that the number systems we use are structured like any mathematical system, regardless of the nature of its elements."[17] An attempt has been made in UMMaP to improve and streamline vocabulary and usage of terms to increase precision and clarity of expression. Mathematics is presented as a language.[18] Keedy has said that:

The primary goal of the project has been to determine

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[15] _Ibid._, pp. 76-77.

[16] _Ibid._, p. 84.

[17] Helen L. Garstens, M. L. Keedy, and John R. Mayor, "University of Maryland Mathematics Project," _Arithmetic Teacher_, 7:63, February, 1960.

[18] _Ibid._, p. 62.

> maturity levels at which certain mathematical concepts can be appropriately taught, and to prepare course materials for a teaching sequence in grades seven and eight that is mathematically and psychologically sound, and appropriate to modern-day needs.[19]

The UMMaP program makes no artificial distinction between algebra and arithmetic. The program attempts to have the student understand mathematics rather than simply learn by rote a variety of mathematical skills.[20] Discovery, both inductive and deductive, is emphasized, and students are led to very informal--but gradually more and more formal--deductive reasoning.[21]

The School Mathematics Study Group (SMSG), formed in 1958, advocates and is attempting to produce an improved mathematics curriculum that would offer students a deeper understanding of the basic concepts and structure of mathematics and that would attract and train more of those students capable of studying mathematics with profit.[22] The SMSG believes that greater substance and interest should be given to the mathematics of grade seven and eight, including ". . . a sound intuitive basis for the algebra

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[19] M. L. Keedy, "Mathematics in Junior High School," Educational Leadership, 17:157, December, 1959.

[20] Ibid., pp. 158-159.

[21] Ibid., p. 159.

[22] School Mathematics Study Group, Newsletter No. 6, March, 1961, (New Haven: Yale University, 1961), pp. 3-4.

and geometry courses to follow."[23] Wagner wrote the following statement about the SMSG materials:

> Emphasized in the texts for junior high school are the following important ideas of junior high school mathematics: structure of arithmetic from an algebraic viewpoint; the real number system as a progressing development; and metric and non-metric relations in geometry. These ideas are constantly associated with their applications. Time is given to the topics of measurement and elementary statistics. Careful attention is paid to the appreciation of abstract concepts, the role of definition, development of precise vocabulary and thought, experimentation, and proof. Materials are chosen with the intent to capture the fascinating features of mathematics, creation and discovery, rather than just utility alone.[24]

The materials of the curriculum reform group appear to not only follow the spirit of "modern" mathematics outlined in the previous section, but they give depth and substance to that spirit. The Commission on Mathematics, though not really engaged in the production of mathematics text materials, has been very influential in guiding progress in curricular reform. The next section outlines the Commission's recommendations for grades seven and eight.

The Recommendations of the Commission on Mathematics. Even though the primary concern of the Commission on Mathematics of the College Entrance Examination Board is

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[23] _Ibid_., p. 4.

[24] John Wagner, "The Objectives and Activities of the School Mathematics Study Group," _The Mathematics Teacher_, 53:455, October, 1960.

for the mathematics of grade nine and above, it did state in 1959 that certain subject matter is essential in grades seven and eight to give students the proper foundation for the Commission's program for grades nine to twelve. Included in its recommendations were the study of fundamental operations, numeration, and ratio in arithmetic. In these areas, students should be led to master the four fundamental arithmetic operations, to understand systems of numeration with place value, to use binary and other notations to strengthen their understanding of decimal notation, to have a knowledge of the arithmetic mean and square root, and to understand ratios as used in comparing quantities of like kind, in proportion, in per cent, and in making scale drawings. With regard to measurement, students should be able to operate with and transform the several systems of measure, to make geometric measurements, to distinguish between the process of measuring and the measure of a quantity, to apply measurement to practical situations, to make scale drawings, and to find length indirectly. Grade seven and eight students should be aware of relationships among geometric elements such as lines, angles, and triangles and of the ideas of symmetry about a point or a line. Students should be able to use line segments and areas to represent numbers and to read and construct graphs. They should be introduced to algebraic concepts such as the use

of placeholders and the concept of a variable as used in simple expressions and sentences.[25]

The views of a number of writers not directly associated with the groups advocating curricular reform, but who have made some pointed recommendations, are presented in the next section to round out the consideration of the nature of current recommendations concerning the teaching of mathematics.

The Recommendations of Various Writers. Nielsen has said that mathematics must be taught: as a cohesive structure; as a logical system built on basic principles, properties, and definitions; and with emphasis on mathematical thinking and logical reasoning. Subjects and topics typically taught as though they were unrelated must be unified. Students must be led to understand basic principles, logical structure and generalized computational procedures so that they will be able to cope with future problems which can't be imagined today. He has also said that the basic concepts, generalizations and skills are the same for slow and gifted students alike, pointing out the universality of "good mathematics."[26]

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[25] Commission on Mathematics, Program for College Preparatory Mathematics (New York: College Entrance Examination Board, 1959), pp. 18-20.

[26] Ross A. Nielsen, "Evaluating the Secondary Math-

Isaacs, in reporting on Piaget's work, has maintained that the unity of arithmetic and mathematics with the rest of one's intellectual life and growth can only be preserved if children are led at an early age to form their own inward structure of number ideas and if the later rule-learning, operation-learning, and the like, become living grafts on that inward structure, successfully developing as a further stage of the inward growth. He has asked if we can learn to carry most children in full understanding and lively interest as we lead them to explore the world of numbers.[27]

Johnson, recognizing the uncertainty of what mathematics our students will need in the future, has said, in part, "that the mathematics taught should emphasize flexibility, procedures, broad principles, and structures rather than minor facts and skills. It should provide experiences in applying concepts and skills, as well as present the aesthetic aspects of mathematics. It should provide experiences that develop good learning habits, as well as a desire to learn."[28]

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ematics Program," The North Central Association Quarterly, 38:250-251, Winter, 1964.

[27] Nathan Isaacs, New Light on Children's Ideas of Number, The Work of Professor Piaget (London: Spottiswoode, Ballantyne, and Co., Ltd., 1960), p. 38.

[28] Donovan A. Johnson, "Evaluating a School Mathematics Curriculum," School and Society, 90:424, December, 1962.

With the emergence of improved curricular materials and recommendations of the nature outlined in the preceding paragraphs, one would expect a variety of reactions from various quarters of the educational realm. The following section contains a sampling of the reactions that have been expressed.

Reactions to Current Curricular Recommendations

Reactions to Excessive Formalism. The literature contains numerous articles warning against the trend towards excessive rigour and formalism in some of the new text materials. However, no specific charges against specific sets of materials were found. Nevertheless, recognizing that an overly formalistic "modern" program could be built that would be just as undesirable as any traditional program with respect to rote learning, the investigator has included in the following paragraphs some of the warnings that have been expressed regarding pitfalls in the excessive use of formalism in school mathematics.

The noted psychologist, Bruner, made his point as follows:

> It is good that a student know how to check the conjecture that $8x$ is equivalent to the expression $3x + 5x$ by such a rigorous statement as the following: "By the commutative principle for multiplication, for every x, $3x + 5x = x3 + x5$. By the distributive principle, for every x, $x3 + x5 = x(3 + 5)$. Again by the commutative principle, for every x, $x(3 + 5) =$

> $(3 + 5)x$ or $8x$. So, for every x, $3x + 5x = 8x$." But it is hopeless if the student gets the idea that this and this only is really arithmetic or algebra or "math" and that other ways of proceeding are really for non-mathematical slobs. Therefore, "mathematics is not for me."
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> . . . .
>
> I would suggest, then, that it is important to allow the child to use his natural and intuitive ways of thinking, indeed to encourage him to do so, and to favour him when he does well.[29]

While conceding that teachers will probably be more enthusiastic in their teaching if they are told the new programs are the key to understanding all mathematics, one writer has expressed the opinion that set theory, questions of the nature of number, excessive rigour, and formal logic (as distinguished from logic in action) are not worth teaching because they are divorced from the mainstream of scientific progress and activity. In his view, better teaching of traditional mathematics would be more useful.[30]

Granting that the new mathematics programs are designed to challenge the intellect, to suit contemporary mathematical thought, and to develop meaning to a much greater extent than programs of twenty years ago, Fehr has shown concern over the amount of formalism and symbolism

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[29] Jerome S. Bruner, "On Learning Mathematics," The Mathematics Teacher, 53:615, December, 1960.

[30] R. J. Diamond, "A Commentary Inspired by the New Mathematics Programs," School Science and Mathematics, 63:663, November, 1963.

presented to very young children, over the hasty creation of specialized teaching materials, and over the rush of administrators, supervisors and teachers to jump on the "new mathematics" bandwagon. He has said that elementary and secondary school education is intended for all-encompassing intellectual development of the child, regardless of his eventual place in life, and it is not aimed directly or solely at producing future mathematicians.[31] Mathematics as a study of formal structure does not make sense unless one has an abundance of experience in developing and applying basic concepts. The formalistic-axiomatic study of mathematics does not provide a medium in which freedom of mind and spirit of adventure can be developed.[32] If we were to stop viewing mathematics as a ". . . sacrosanct monument having eternally true hypotheses and theorems arranged in immutable order . . ." and place emphasis on discovery and creative ability, mathematics instruction would flourish and pupils could not be shunted away from the subject.[33]

Piaget contends that "when adults try to impose

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[31] Howard F. Fehr, "Modern Mathematics and Good Pedagogy," Arithmetic Teacher, 10:402, November, 1963.

[32] Ibid., p. 404.

[33] Ibid., p. 405.

mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth."[34] He views the effective teacher as one who aids the child's natural development of mathematical concepts by supplying a rich environment for discovery.

Supposing that textbook materials can be found that are not excessively formalistic, how should teachers present these materials?

Reactions Dealing with the Teaching Approach. Kline, noting that there is universal agreement that the present mathematics curriculum is poor, has said that the trouble lies in the approach to the material rather than in the material itself.[35] Somewhat along the same line of thinking, Read has said:

> We must realize that something is not wrong merely because it is old or traditional. Some traditional material still has great value.
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> . . . .
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> We wish to teach more mathematics more creatively, to teach the true nature of mathematical thought; we must supply motivation and purpose; in other words, creative teaching . . . [36]

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[34] Jean Piaget, "How Children Form Mathematical Concepts," Scientific American, 189:74, November, 1953.

[35] Morris Kline, "Reform in the Mathematics Curriculum," Education Digest, 24:36, February, 1959.

[36] Cecil B. Read, "New Wine in Old Bottles," School Science and Mathematics, 61:165, March, 1961.

"In looking at the content of the new programs for grades 9-12," Beberman, the director of the University of Illinois Committee on School Mathematics (UICSM), has said, "I am impressed more by the attempt to organize the traditional subject matter along logical lines than by the inclusion of new subject matter."[37] According to Bebermann:

> Objections to the new programs should not center on questions concerning new subject matter. The real question is whether or not students should understand the skills which are taught in both the conventional and the new programs.
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> . . . .
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> [However, one should avoid] . . . the terrible hazard of thinking that any approach which emphasizes logical explanations leads to understanding. . . . . the trick is to build a curriculum in which the mathematics is logically developed and to match this curriculum both to the present interests, needs and capacities of the learners, and to their future interests and needs.[38]

Delon has warned that some teachers are presenting new content, which has been introduced to help unify and clarify mathematics, in almost complete isolation from other phases of mathematics instruction. Some teachers fail to realize that the intrinsic value of the new content is secondary to an over-all understanding of the mathematical structure it is intended to convey, and they substitute

. . . memorization of the new terminology for the

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[37] Max Beberman, "The Old Mathematics in the New Curriculum," Educational Leadership, 19:373, March, 1962.

[38] Ibid., p. 375.

> memorization of traditional definitions and rules of computation. On the elementary school level, the result of requiring the use of labels and terminology beyond the comprehension of the child is to deter rather than facilitate his learning.[39]

Evidently, as one would expect, a great deal depends on what the teacher does with the materials. The best program possible would be of no avail if it were not used in the spirit intended. It seems to follow that teacher (and student) attitudes would have some bearing on the effectiveness of any program.

Teacher and Student Reaction. One need not read far in the literature before grasping what the student and teacher reactions to the new programs are. A few of the citations typical of those found throughout the literature are presented in succeeding paragraphs.

> Most teachers who have used the new mathematics programs are enthusiastic and say they will never return to the old. Students enjoy the work and show far more understanding.[40]

> Teachers all over the country have commented to me that their students are showing more interest and excitement in the study of the new mathematics programs than ever before. Even the weak students in mathematics appear to be liking mathematics.[41]

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[39] Floyd G. Delon, "The Aftermath of a Revolution," Arithmetic Teacher, 10:482, December, 1963.

[40] Howard F. Fehr, "A Great Step Forward," Education Digest, 28:42, January, 1963.

[41] W. Eugene Ferguson, "Current Reforms in the Math-

It has been found that some students who apparently had received A's and B's in traditional mathematics on the strength of their ability to memorize and compute, have experienced real difficulty with programs calling for understanding and abstract reasoning, while students labelled as "poor in mathematics" have come alive when released from routine drill and given freedom of thought. Furthermore, college students have frequently reported that SMSG or UICSM high school courses have better prepared them for college than traditional courses have prepared their friends.[42]

After interviewing SMSG teachers, one writer was led to report: "The SMSG units are stimulating to teach and the material tends to be self-motivating."[43] Marvelling at the enthusiasm of students (and parents!), he anticipated the creation of fewer arithmetic "haters" and more good thinkers from the use of such programs.[44]

Somewhat in the same vein, another writer revealed

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ematics Curricula--A Passing Phase or Progress?" The Mathematics Teacher, 57:145, March 1964.

[42] Ibid.

[43] Margaret F. Willerding, "A Critical Look at the New Mathematics for Seventh Grade," School Science and Mathematics, 62:218, March, 1962.

[44] Ibid., p. 220.

that:

> . . . the reaction of children to new mathematics has been overwhelmingly positive. They are truly fascinated by these new approaches. Classes have been so intrigued that they wanted to forego gym in order to continue a mathematics lesson.[45]

Commenting on their experience with the Ball State Program, Brumfiel, Eicholz, and Shanks have reported that teachers with strong mathematics backgrounds are enthusiastic about the experimental program, but teachers with weak backgrounds have mixed reactions, although most feel that as they develop the required skills and understandings they will be able to teach experimental material more effectively than traditional material.[46]

Indicating a concern frequently expressed in reference to many of the new programs, an editor of the Arithmetic Teacher raised the following question: "Within the limits of computation, can the UMMaP students add, subtract, multiply and divide with whole numbers and common and decimal fractions satisfactorily? Can they use these computations in percentage and mensuration?"[47]

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[45] L. B. Fisher, "How Curriculum Builders View New Mathematics Ideas," School Science and Mathematics, 64:34, January, 1964.

[46] Charles Brumfiel, Robert Eicholz, and Merrill Shanks, "The Ball State Experimental Program," The Mathematics Teacher, 53:76, February, 1960.

[47] Ben A. Sueltz, "Editor's Note," Arithmetic Teacher, 7:65, February, 1960.

On the whole, however, the reactions of students and teachers alike seem to be very enthusiastic. Such a response is likely to have a beneficial effect on the learning of mathematics.

In the preceding paragraphs, an overview of the driving forces, characteristics, and effects of the movement towards improved school mathematics has been presented. It might be asked how well the two new commercial grade eight mathematics texts that were used in the experiment carried out in Alberta schools in 1963-64 reflect this movement. Some indication of what the answer might be is given in the following section.

Reviews of STM and EMM Texts

Four paraphrased, teacher-written reviews of STM and EMM texts for grades seven and eight are presented in the succeeding paragraphs.

Seeing Through Mathematics, Book 1. In this text mathematical terms are defined in dictionary style. Material is included on sets, numeral-number contrasts, numeration systems, geometry, measurement, reading of mathematical symbols, solution of conditions using the set approach, rate pairs, per cent, and business arithmetic. The text is student-centered with the development primarily in the form of questions framed for the student to answer. Each lesson

facilitates student anticipation of what is to be learned and helps the pupil to summarize the main ideas developed in the lesson.[48]

Seeing Through Mathematics, Book 2. As in Book 1, the guided discovery method is used. The treatment of rational numbers is strictly mathematical and lays a foundation for operations involving positive and negative numbers and zero. The real number system and its relation to the number line is studied. The concept of measure as an interval is developed, leading to consideration of precision and accuracy. Conversion of units and consideration of triangle properties are based on the rate-pair concept. Equivalence conditions are used to derive formulas from geometry, science, business, industry, and mathematics. The development of plane and space geometries is consistent. The text concludes with a unit on probability and statistics.[49]

Exploring Modern Mathematics, Book 1. This text lays

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[48] Francis C. Theaker, Seeing Through Mathematics, Book 1, a book review in "Reviews and Evaluations," edited by Kenneth B. Henderson, The Mathematics Teacher, 56:640-641, December, 1963.

[49] Francis C. Theaker, Seeing Through Mathematics, Book 2, a book review in "Reviews and Evaluations," edited by Kenneth B. Henderson, The Mathematics Teacher, 56:641-642, December, 1963.

the foundation for future study of mathematics in such a way that the student can identify with "the excitement, secrets, and practicality of great mathematicians." The number-numeral distinction is made in a historical context. Primes, factors, divisors, greatest common factors, least common multiples, the fundamental theorem of arithmetic, and all the field axioms are covered in the development of number concepts. The notation and language of sets is used to develop geometric concepts and terminology such as betweenness, half-line, and convex polygon. Subtraction and division are presented as the inverse operations of addition and multiplication. The treatment of inequalities and equalities is facilitated by using the language of sets. Manipulation of symbols is de-emphasized. Modular arithmetic is presented as a first example of a mathematical system, and the concept of a mathematical system is used to present operations with fractions in a meaningful light. The treatment of measure, built on the concept of an arbitrary basic unit, is consistent for one- and two-dimensional figures. Towards the end of the text much of the traditional type of seventh grade arithmetic is presented.[50]

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[50] Mary Laycock, Exploring Modern Mathematics, Book 1, a book review in "Reviews and Evaluations," edited by Kenneth B. Henderson, The Mathematics Teacher, 56:546-547, November, 1963.

Exploring Modern Mathematics, Book 2. The integration of arithmetic, algebra and geometry is attained in this text through: many algebraic and geometric topics scattered throughout the book, a chapter on the solution of algebraic equations, a chapter on polynomials, and two chapters on geometry. The extent to which rigour gradually increases through the text, as the authors claim it does, is difficult to assess, but, in the reviewer's opinion, not enough is done to prepare students for the writing of formal proofs, which is treated in the third book of the series. There is increasing emphasis on applied problems and increasing attention to reasoning throughout the text. Opportunity for the student to explore and discover for himself, one of the better features of the text, is facilitated through well-placed Let's Explore exercises. Strategically located skill maintenance drills are employed often. The development of number systems is similar to that used in SMSG. According to the reviewer, in a few cases structure is sacrificed for the sake of expediency in the introduction of new material.[51]

It would appear from the reviews that each of the

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[51] Sol Mastbaum, Exploring Modern Mathematics, Book 2, a book review in "Reviews and Evaluations," edited by Kenneth B. Henderson, The Mathematics Teacher, 57:34, January, 1964.

STM and EMM texts for grades seven and eight is written in the spirit of current curricular recommendations.

III. REVIEW OF RESEARCH ON THE EVALUATION OF NEW MATHEMATICS MATERIALS

According to Mayor, all of the larger projects involved in producing improved mathematics curricula have provided for classroom try-out of their materials, for reports from teachers using the textbooks, and for modification of the programs based on teacher reports. Even though, in most instances, there has not been controlled research, Mayor has pointed out that the fact that materials are being tried out in classrooms with revisions being made on the basis of classroom experience is a healthy development in the area of school curriculum design.[52]

In what follows, the investigator has summarized the findings of recent research studies particularly concerned with the evaluation of student achievement in new mathematics courses.

Payette[53], of the Educational Testing Service (ETS),

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[52] John R. Mayor, "Research in Review--Mathematics Education," Educational Leadership, 19:395-396, March, 1962.

[53] Roland F. Payette, "Educational Testing Service Summary Report of the School Mathematics Study Group Curriculum Evaluation," Newsletter No. 10, November, 1961 (Stanford, California: Leland Stanford University, 1961), pp. 5-11.

has reported a study in which student achievement in SMSG courses was compared with student achievement in non-SMSG courses. In the summer of 1960, the ETS made arrangements with approximately seventy-five schools to have them participate in a major experimental study designed to provide evidence pertinent to the following three questions:

> 1. Does the SMSG curriculum detract from student achievement with respect to traditional "mathematical skills"?
> 2. Does the SMSG curriculum result in a measurable extension of developed mathematical ability beyond that of conventional mathematics instruction?
> 3. How effectively is the SMSG curriculum communicated to students at various levels of scholastic ability?[54]

At each of the seventh, ninth, tenth, eleventh and twelfth grade levels, thirty teachers, selected at random from a group of teachers willing to teach SMSG for the first time, provided their students with conventional mathematics instruction, and thirty teachers, selected at random from the aforementioned group, provided their students with mathematics instruction based on SMSG materials. In the fall of 1960, all students involved in the study were given common tests of scholastic aptitude and knowledge of mathematics. In the spring of 1961, all of the students were given common tests of conventional mathematics and of SMSG materials.

On the basis of data collected from the administration

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[54] *Ibid.*, p. 5.

of conventional mathematics tests and in consideration of initial student aptitude and knowledge of mathematics, Payette reported that ". . . over-all, students exposed to conventional mathematics have neither a pronounced nor a consistent advantage over students exposed to SMSG mathematics with respect to the learning of traditional mathematical skills."[55] The same conclusion was reached when the achievement of the above-mentioned conventional mathematics students was compared with that of SMSG mathematics students taught by teachers selected at random from a group who _had_ taught SMSG mathematics previously.

Furthermore, on the basis of data collected from the administration of SMSG tests and in consideration of initial student aptitude and knowledge of mathematics, it was found that ". . . students exposed to SMSG instruction acquire pronounced and consistent extensions of developed mathematical ability beyond that developed by students exposed to conventional mathematics instruction."[56] In addition, positive evidence was found that suggested that students at all scholastic ability levels can learn considerable segments of SMSG materials.

Another study involving SMSG materials has been

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[55] _Ibid_., p. 7.

[56] _Ibid_., p. 10.

reported by Rosenbloom.[57] In this study, which was carried out during the 1959-60 and 1960-61 school terms, random selections were made from a population of Minnesota teachers who had volunteered for experimental work. Both well and poorly qualified teachers from large cities, small towns, and rural areas were represented. At each of the grade levels from seven to twelve, between 100 and 400 students were involved in the 1959-60 study, and between 300 and 400 were involved in the 1960-61 study. In addition, sixty grade six students who were studying grade seven SMSG materials were involved in both studies.

Each September, the Differential Aptitude Tests (DAT) for grades nine through twelve and the School and College Ability Tests (SCAT) for grades seven and eight were administered to measure student aptitude. Each September, the following May, and the following September, the Sequential Tests of Educational Progress (STEP) appropriate to each grade level were administered to measure achievement with respect to goals common to both SMSG and conventional mathematics curricula.[58]

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[57] Paul Rosenbloom, "Minnesota National Laboratory Evaluation of SMSG, Grades 7-12," Newsletter No. 10, November, 1961 (Stanford, California: Leland Stanford University, 1961), pp. 12-26.

[58] *Ibid.*, p. 13.

In a particularly well-controlled phase of the experiment, grade seven students taking the SMSG course scored significantly higher than those in control classes on the May, 1960 STEP post-test. However, on the September, 1960 retention test, the SMSG students scored only slightly higher than the control students. At the grade seven level it was also found ". . . that the low-ability students profit more from the SMSG course, in comparison to the conventional, than the other pupils, and that the course does not place an undue strain on their verbal ability."[59] Furthermore, evidence suggested that teachers need not be specially trained or highly qualified to teach the SMSG seventh grade course successfully.[60]

No control classes were established at the sixth, eighth, ninth, tenth, eleventh or twelfth grade levels. Consequently, comparisons could only be made on the basis of test publisher's data on the STEP performance of students in conventional classes. The children who studied seventh grade SMSG materials for a complete term in grade six were found to compare favourably with eighth grade students, nationally. No ability group of SMSG students in any of the grades eight through twelve was found to do much worse on

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[59] *Ibid.*, p. 23.

[60] *Ibid.*, p. 24.

the STEP tests than would be expected according to national norms. "In all cases the students did as well as would be expected, and in some cases much better."[61]

Regarding the correlation between teacher qualifications and student achievement, for which the experimental analyses gave no significant results, Rosenbloom made the following statement:

> We should not like to encourage misinterpretation of our results on the relations between teacher qualifications and student achievement. We do not believe that subject-matter competence is irrelevant to a teacher's effectiveness. Our evidence merely indicates that the usual ways of measuring this competence, in terms of experience, credit hours, and course grades, are not satisfactory.[62]

Williams and Shuff[63] have reported on the results of experimental studies, carried out by them during the 1960-61 school term, which involved conventional and SMSG mathematics students in the Roseville School District in Minnesota. Their report is based on two unpublished doctoral dissertations, one completed by Williams[64] and the

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[61] Ibid.

[62] Ibid., p. 25.

[63] Emmet D. Williams and Robert V. Shuff, "Comparative Study of SMSG and Traditional Text Material," The Mathematics Teacher, 56:495-504, November, 1963.

[64] Emmet D. Williams, "Comparative Study of SMSG and Traditional Mathematics" (unpublished Doctoral Dissertation, The University of Minnesota, Minneapolis, 1962)

other completed by Shuff.[65] The general question for which answers were sought in these two studies was:

> In what way do students who have studied experimental text materials in mathematics differ from students who have studied traditional materials?[66]

The seventh grade sample was made up of eight classes (216 students), established through line scheduling. Each of four teachers taught one section with SMSG materials and one with traditional materials.[67]

The eighth grade sample was chosen by random sampling from students who scored above the fiftieth percentile on the Snader General Mathematics Test and who obtained a "C" or better in the previous mathematics course. The six classes (172 students) involved were randomly assigned SMSG or traditional treatment. Each teacher taught one SMSG class and one traditional class.[68]

The grade nine sample was made up as follows: one class with SMSG background and one with traditional mathematics background, both of which studied the SMSG grade

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[65] Robert Vance Shuff, "A Comparative Study of Achievement in Mathematics at the 7th and 8th Grade Levels Under Two Approaches, School Mathematics Study Group and Traditional" (unpublished Doctoral Dissertation, The University of Minnesota, 1962).

[66] Williams and Shuff, op. cit., p. 495.

[67] Ibid., p. 496.

[68] Ibid.

nine course; one class with SMSG background and one with traditional mathematics background, both of which studied the traditional grade nine mathematics course; and two classes that were a mixture of students with SMSG or traditional background, one of which studied the grade nine SMSG course while the other studied the traditional grade nine course.[69]

At the tenth grade level, about forty SMSG students who scored above the 90th percentile on publisher's norms on the Differential Aptitude Test (DAT) were matched with traditional mathematics students on the basis of DAT scores.[70]

A listing of the tests used in the experiment follows:

Grade Level	Scholastic Aptitude	General Mathematics Achievement	Specific Mathematics Achievement
7	SCAT	STEP 3A,3B	COOP Mathematics X,Y
8	SCAT	STEP 3A,3B	COOP Mathematics X,Y
9	DAT	STEP 2A,2B	COOP Elementary Algebra T,Y
10	DAT	STEP 2A,2B	COOP Intermediate Algebra T,Y

The STEP tests and COOP tests were described as conventional, pre-SMSG tests of mathematical achievement.[71]

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[69] _Ibid_., p. 497.

[70] _Ibid_., pp. 497-498.

[71] _Ibid_., p. 498.

At the seventh, ninth and tenth grade levels, experimenters found no significant differences between mean scores obtained by the SMSG classes and mean scores obtained by the traditional mathematics classes on any of the conventional standardized mathematics tests used in the experiments. However, ". . . at the eighth-grade level, the advantage seemed to be with the traditional classes; results differed with a statistical significance of .05 on STEP and .01 on COOP."[72]

The experimenters recognized that a test related to the content and objectives unique with the SMSG materials would be needed before a complete evaluation of the effectiveness of such materials could be made.[73]

In reporting a 1960-61 study that involved four accelerated seventh grade mathematics classes (two on a "modern" program and two on a conventional program), Ruddell[74] reached essentially the same conclusions as those stated for each of the studies so far reviewed in this chapter. Ruddell administered three traditional mathematics tests to the four classes at the end of the school term.

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[72] Ibid., p. 504.

[73] Ibid.

[74] Arden K. Ruddell, "The Results of a Modern Mathematics Program," Arithmetic Teacher, 9:330-335, October, 1962.

These were the Wide Range Achievement Test (WRA) in Arithmetic, a STEP test, and a paper-and-pencil test prepared by the investigator to evaluate traditional mathematics understandings. It was found that, on all three tests, the students who studied the "modern" program scored as high as or higher than children who studied the traditional mathematics program.[75]

In the 1961-62 school term, Nelson conducted an experiment in which he sought an answer to the question: ". . . does simplification of the textbook presentation of mathematics material have an effect upon the mathematics achievement of high ability junior high school pupils?"[76] Twelve seventh grade classes and sixteen ninth grade classes participated in the experiment. The classes at each grade level were paired on the basis of past mathematics achievement and general ability, and one teacher taught each pair, using an SMSG text written for college capable students with one class, and using an SMSG text written for slower learning pupils with the other. At either grade level, the two versions of the SMSG text covered essentially the same content, but in the modified text simpler presentations

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[75] _Ibid_., p. 335.

[76] L. Doyal Nelson, "Relation of Textbook Difficulty to Mathematics Achievement in Junior High School" (unpublished Doctoral Dissertation, the University of Minnesota, Minneapolis, 1962), p. 42.

were used and the reading difficulty was reduced. The grade seven students involved in the experiment were identified as being from the top half of the ability range in their school. The ninth grade students were from the top third of the ability range.[77] At the beginning of the experiment, September 1, 1961, STEP mathematics tests were administered to all students. During the course of the experiment all students wrote appropriate SMSG unit tests and either a SCAT test (grade seven) or a DAT test (grade nine). In May, 1962, alternate forms of the STEP test were administered.[78] The statistical analysis of scores obtained on the various tests included analysis of covariance procedures and examination of regression lines for trends.[79]

At the seventh grade levels, the experimental results led the investigator to conclude:

> Since the modified text seemed to favor low achievers in the high ability group involved, there can be little doubt that the kinds of changes SMSG made in producing the M [modified] texts tend to lead to greater efficiency in the learning of mathematics for all but the highest achieving group of pupils in a school.[80]

Similar findings were reported on the basis of results from the ninth grade sample.

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[77] _Ibid_., pp. 7-8.

[78] _Ibid_., p. 75.

[79] _Ibid_., pp. 77-80.

[80] _Ibid_., p. 170.

An experiment involving grade seven commercial mathematics text materials was carried out by the Alberta Junior High School Mathematics Subcommittee during the 1962-63 school term. Data were obtained for 253 students studying STM, Book 1; 171 students studying EMM, Book 1; and 184 students studying *Winston Mathematics, Intermediate Book 1*. Analysis of the scores obtained by the students in the three different instructional groups on a STEP mathematics test administered in May, 1963 indicated that there were no significant differences among the groups with regard to achievement as measured by a conventional mathematics test. However, on a special test devised by members of the Subcommittee to measure student understanding of content in the EMM and STM text materials, there were statistically significant differences found among the mean scores obtained by the groups of students that studied the different programs. Except at the low ability level (as indicated by SCAT Total Scores), the students who studied EMM materials scored higher on the special test than those who studied STM materials (The differences were significant at the .05 level.). At all ability levels the STM and EMM students scored significantly higher than the *Winston Mathematics* students on the special test.[81]

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[81] Alberta Junior High School Mathematics Subcommittee,

Alpert, Stellwagon, and Becker[82] have reported a major investigation of many of the psychological variables relevant to seventh grade mathematics, "modern" and conventional. Their sample consisted of approximately 270 subjects, half boys and half girls. Half of the sample consisted of students from SMSG classes, and the other half consisted of students from conventional mathematics classes.[83] Among other things, the results from the study indicated ". . . that the experimental program does not increase students' positive feelings toward mathematics, either absolutely or relative to the traditional mathematics program."[84] However, it was found that theoretically oriented teachers produced positive student feelings towards mathematics in SMSG classes but not in non-SMSG classes. Another interesting finding was that:

> . . . at the start of the school year, the SMSG students were found to be more favorably oriented toward mathematics than the non-SMSG students. However, the data show that at the end of the school year . . . while non-SMSG mathematics attitudes remained relatively

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"Interim Report on Testing Program, June, 1963." (Mimeographed.)

[82] R. Alpert, G. Stellwagon, and D. Becker, "Psychological Factors in Mathematics Education," Newsletter No. 15, April, 1963 (Stanford, California: Leland Stanford University, 1963), pp. 17-24.

[83] *Ibid*., p. 17.

[84] *Ibid*., p. 23.

> constant, SMSG students' attitudes fell. The general enthusiasm was gone.[85]

The latter finding was taken as an implication that ". . . the job now is to modify further the material and to _train_ _teachers_ in a way that will justify the students' initially high expectations."[86]

IV. CHAPTER SUMMARY

Reviews from the literature were included in this chapter to present a picture of the environment in which present-day considerations regarding the adoption of new school mathematics curricula must proceed. In the succeeding paragraphs the writer attempts to summarize the salient views and findings reported in the chapter.

The case for reforming school mathematics curricula seems very strong, indeed, when one reflects upon the reported weaknesses of many traditional mathematics students; upon the unstructured, frequently boring nature of traditional mathematics textbook materials; and upon the vast technological revolution occuring in our times.

New or "modern" school mathematics is widely identified as that which places emphasis on student discovery

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[85] _Ibid_.

[86] _Ibid_., p. 24.

and understanding of basic principles, on the introduction of unifying concepts, on precise expression, and on an imaginative approach to the subject. This view is substantiated by the various experimental programs for the seventh and eighth grades, by the recommendations of the Commission on Mathematics, and by the views of various writers active in mathematics education.

Regarding new school mathematics curricula, the most frequently noticed reaction from writers in the field of mathematics education is the warning that the good in the new programs might be spoiled by an attempt to be overly rigorous or formalistic. It is widely agreed that intuitive understanding and appreciation of concepts must occur well before any attempts are made to develop highly symbolized, formalistic-axiomatic mathematics with students.

Another frequently raised and well-taken point is that, if the new materials are to be effective, the teacher's presentation of them must be in harmony with the spirit in which they were written. Nothing will be gained if teachers present new materials so as to encourage learning by rote as has been common in the past.

In general, students and teachers alike have been found enthusiastic about the new mathematics materials. Possible exceptions noted have been teachers with deficiencies in their mathematics background who at first have

expressed negative attitudes toward the reforms. However, after teaching and learning from the new materials, apparently even these teachers have become enthusiastic.

According to teacher-written reviews of the STM and EMM texts for grades seven and eight, these commercially produced textbooks do reflect the spirit of the curricular recommendations of the national school mathematics reform groups.

Research studies that have evaluated the effectiveness of new mathematics curricula on the basis of student achievement have been consistent in their findings, even though a tendency was observed for reporters of research to interpret neutral findings with a slight bias, which seemed to depend on the views of the researcher. In general, it was found that:

1. Students studying "modern" mathematics materials and students studying traditional mathematics materials perform equally well on conventional mathematics tests.
2. Whenever a test reflecting the spirit and objectives of "modern" mathematics curricula is included in a study, the "modern" group scores much higher than the traditional group.
3. The new curricular materials appear to be appropriate for all levels of scholastic ability, and indications are that they may give an added advantage to lower ability students.

One reporter of research indicated that the usual measures of teacher competence (experience, university course grades, and the number of mathematics courses taken) seemed, by and large, unrelated to student achievement.

One researcher found that there may be certain advantages in using simplified but honest and complete presentations of school mathematics rather than more sophisticated approaches.

A research study on psychological factors associated with the learning of seventh grade mathematics found that initial high interest in new mathematics materials drops off toward the end of the first year. Indications were given that this could be remedied through revised materials and better teacher education.

CHAPTER III

THE EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

Do grade eight students develop any greater understanding of basic mathematical ideas, in the areas described in the "Guidelines,"[1] if they study STM or EMM text materials rather than the Winston Mathematics text? Do students score as well on a conventional arithmetic test if they study STM or EMM text materials rather than the Winston Mathematics text? How do STM students compare with EMM students with respect to their understanding of basic mathematical ideas and on the basis of scores obtained by them on a conventional arithmetic test? The major purpose of the study presently being reported was to produce evidence that might provide a basis for answering these questions.

The present chapter contains a detailed description of the nature of the components of the experiment, the design of a special mathematical understandings test, the design of the experiment, and the statistical procedures followed in processing the data.

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[1] See Chapter I, pp. 4-5.

I. THE NATURE OF THE SAMPLE

During the 1963-64 school term, twenty-two classes of grade eight students, from the two major cities and three of the towns in the province of Alberta, participated in the experiment. Eleven of these classes, referred to as the STM group, studied portions of Seeing Through Mathematics, Book 1 and Seeing Through Mathematics, Book 2. Five classes, referred to as the EMM group, studied portions of Exploring Modern Mathematics, Book 1 and Exploring Modern Mathematics, Book 2. The remaining six classes, referred to as the CON group, studied Winston Mathematics, Intermediate 2, the currently authorized, conventional mathematics text for grade eight.[2] There were six STM teachers, four EMM teachers, and six CON teachers involved in the experiment.

The participating teachers and students were selected by members of the Alberta Junior High School Mathematics Subcommittee from schools in the Subcommittee members' respective school districts. Ideally, it would have been desirable to have made a random assignment of pupils, teachers, and treatments, but since the study was conducted in a practical school situation and since one determining factor was the availability of teachers willing to

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[2] See Chapter I, pp. 6-7 for detailed footnote references to these text materials.

participate in the experiment, such an assignment was not possible. However, examination of Table I on page 60 would lead one to conclude that the three groups of students studying the different texts were fairly well matched as to scholastic ability.[3] Similarly, consideration of evidence collected on the characteristics of teachers involved in the experiment[4] would support the contention that the three groups of teachers were comparable in overall training and experience.

However, inspection of Table I shows that roughly one-third of the students in the sample had School and College Ability Test (SCAT) Total scores above the ninetieth percentile on the test publisher's (1957) norms. How the scores of grade eight Alberta students in general would appear in comparison with the norms is a matter of conjecture, but it would seem safe to assume that the sample as a whole evidenced better than average scholastic ability. Nevertheless, since about sixteen per cent of the students in the sample had SCAT Total scores below the fifty-second percentile, there seems to be good reason to postulate that the average and low ability students are

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[3] See also evidence from statistical tests in Chapter III, on page 86.

[4] See Chapter IV, pp. 117-127.

TABLE I

PROPORTIONS OF THE SAMPLE WITH SCAT TOTAL SCORES IN RANGES CORRESPONDING TO MID-PERCENTILE RANKS ON THE TEST PUBLISHER'S NORMS

SCAT Total Raw Score	Mid-Percentile Rank on SCAT Norms*	CUMULATIVE PROPORTIONS			
		Entire Sample	STM Group	EMM Group	CON Group
105-109	99+	.98	1.00	1.01	.98
102-104	97-98	.93	.95	.99	.90
101	96	.86	.87	.94	.85
99-100	94	.83	.83	.92	.82
97- 98	92	.78	.77	.88	.76
95- 96	90	.71	.71	.80	.66
92- 94	86	.64	.64	.73	.58
90- 91	81	.54	.55	.63	.45
87- 89	76	.47	.49	.55	.37
83- 86	71	.42	.45	.48	.32
80- 82	65	.34	.36	.36	.29
76- 79	58	.26	.30	.26	.20
72- 75	52	.16	.20	.16	.10
67- 71	45	.11	.14	.09	.09
63- 66	38	.07	.10	.05	.05
59- 62	32	.04	.06	.04	.02
45- 58	12-26	.02	.03	.02	.01

*Cooperative Test Division of the Educational Testing Service, *Cooperative School and College Ability Tests, Technical Report* (Princeton, N. J.: Educational Testing Service, 1957), p. 40.

fairly well represented in the lower third of the distribution. It would seem reasonable to assume that any conclusions reached regarding students who scored in the lower third of the sample distribution of SCAT Total scores (up to the sixty-fifth percentile) would apply, in general, to grade eight Alberta students of average scholastic ability. Conclusions reached about students with scores in the middle and upper third of the distribution should apply in general to grade eight students with high and very high measurable scholastic ability.

II. THE NATURE OF THE TEXTBOOKS

The reviews of the STM and EMM texts reported in Chapter II[5] give some indication of the nature of these texts, but to supply more information, the investigator has included Table II and Table III, which list the chapter headings for all three sets of text materials used in the experiment. Furthermore, quotations of the textbook authors' views on the nature and objectives of their texts, and examples illustrating the approaches to various topics employed by each of the text materials are presented on the succeeding few pages.

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[5] Pages 36-39.

TABLE II

CHAPTER HEADINGS OF THE STM, EMM, AND CON (WINSTON MATHEMATICS) TEXT MATERIALS FOR GRADE SEVEN

STM, BOOK 1	EMM, BOOK 1	WINSTON MATHEMATICS, INTERMEDIATE 1
1. Sets, Conditions, and Variables	1. Numeration Systems	1. How Well Do You Understand Whole Numbers?
2. Intersection and Union of Sets	2. Properties of Whole Numbers	2. How Well Do You Understand Fractions?
3. Conditions in Two Variables	3. Points, Lines, Curves and Planes	3. How Well Do You Understand Decimal Fractions?
4. Conditions Involving Rate Pairs	4. Factoring and Prime Numbers	4. The Language of Per Cent
5. Numeration Systems	5. Mathematical Systems	5. Graphs and Their Uses
6. The Natural-Number System	6. The Number System of Arithmetic	6. Squares and Rectangles
7. The Rational Numbers of Arithmetic	7. Figures on a Plane	7. Triangles and Circles
8. Geometry and Measurement	8. Measures	8. Mathematics of the Home
	9. Applying the Number System of Arithmetic	9. Mathematics of Business

TABLE III

CHAPTER HEADINGS OF THE STM, EMM, AND CON (WINSTON MATHEMATICS) TEXT MATERIALS FOR GRADE EIGHT

STM, BOOK 2	EMM, BOOK 2	WINSTON MATHEMATICS, INTERMEDIATE 2
9. Sets of Numbers and Sets of Points	1. The System of Integers	1. How Well Do You Understand Whole Numbers and Common Fractions?
10. The Rational Number System	2. Geometry: Congruence, Constructions and Circles	2. How Well Do You Understand Decimal Fractions?
11. The Real Numbers	3. Exponents and Scientific Notation	3. Per Cent and Its Applications
12. Measurement	4. The System of Rational Numbers	4. Areas of Familiar Figures
13. Formulas	5. Geometry: Areas, Regular Polygons and Circle Graphs	5. Finding Simple Interest
14. Plane and Space Geometry	6. More About Integers and Rational Numbers	6. Compound Interest and Savings
15. Permutations, Combinations, and Number Theory	7. Solving Equations	7. Geometric Constructions and Graphs
16. Probability and Statistics	8. Polynomials	8. Solids and Volume
	9. Applied Problems, Conjunctions of Equations	9. Paying for the Cost of Government
		10. Indirect Measurement and Equations

The Authors' Perceptions of the Nature and Objectives of Their Texts

The STM Series

> In keeping with our philosophy of teaching mathematics in a meaningful way, we have used certain basic concepts throughout STM. These basic concepts--the ideas of set, set operation, condition, variable, mathematical structure, relation, logic, and problem solving--serve as "strands" that provide a broad perspective of the field of mathematics. These strands permeate STM, strengthening the presentation of our program and enabling students to view mathematics as a unified whole.[6]
>
> Throughout STM, careful attention is given to the distinction between the symbols used as a means of communication in mathematics and the ideas expressed by these symbols. This distinction between names and referents of the names is an integral part of the entire program.
>
> The ideas of set, conditions, and variables help to give precise meaning to topics that have been presented mechanically in traditional texts.[7]
>
> Remember that STM is not organized around "social" arithmetic. However, "social" arithmetic is included whenever it is appropriate in terms of the mathematical ideas being developed.
>
> It should be clearly evident . . . that there are two major themes in the STM program: (1) understanding of mathematical ideas (not memorization of number facts); and (2) integration of mathematical ideas (not compartmentalization).[8]

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[6] Henry Van Engen and others, Teaching Guide, Seeing Through Mathematics, Book 1 (Chicago: Scott, Foresman and Company, 1963), p. 8.

[7] Ibid., p. 9.

[8] Ibid., p. 13.

The EMM Series

> The series was inspired by and is based upon the work of the University of Maryland Mathematics Project (Jr. H. S.) and the School Mathematics Study Group. The authors have used what they feel to be the best of the mathematical and pedagogical ideas found in these works, and have of course made their own extensions, organization and improvements.
>
> . . . Arithmetic and algebra have been integrated, so as to eliminate the artificial distinction between them, and in order to provide more than one year's time to digest the important ideas of the latter. Geometry is also integrated with other materials where practical, and is to be found throughout the series. Throughout the series mathematical rigor will be found gradually to increase, as well as attention to and practice in working applied problems. Provision has also been made to place gradually more responsibility on the student for reading and learning mathematics by himself.
>
> . . . .
>
> The authors concur with the many who contend that students learn and retain better when they make their own discoveries or gain insights from their own experience, and these books are written accordingly. Exploratory exercises, headed Let's Explore, provide experiences which lead to discovery or insight, and new concepts are almost invariably introduced in this way. It is only later, after this student experience, that the concepts are verbalized or summarized in the text.[9]

> An important feature of this series is that it emphasizes understanding, based on structure. The study of structure is based on the concept of a mathematical system.[10]

> IT IS NOT RECOMMENDED that the teacher should make too much of ultra-precise vocabulary. Student verbalizations are not often terribly precise, and to enforce

---

[9] Mervin L. Keedy, Richard E. Jameson, and Patricia L. Johnson, Exploring Modern Mathematics, Book 1, Teacher's Edition (New York: Holt, Rinehart and Winston, Inc., 1963), p. v.

[10] Ibid., p. xi.

> linguistic precision prematurely is harmful rather than helpful. On the other hand, working toward gradual linguistic maturation is recommended.
>
> . . . The authors have striven to make their language consistent, precise and economical, so that the books will provide help with the linguistic problem when needed.[11]

The Winston Mathematics (CON) Series

No introductory remarks or other statements by the authors about the nature or objectives of this series were found in the texts. However, it appears that the purpose of the series is to teach arithmetic and problem solving skills incidentally in "social" situations.

Treatments of Various Topics by Each Text Series

On the following pages the investigator has presented examples of textbook lessons to illustrate for the reader characteristic differences in the approaches to various topics used by each of the three textbook series involved in the experiment. The examples were selected from introductory material dealing with topics in each of the five areas particularly being considered in the study presently being reported (Measurement, Number Systems, Numeration, Geometry and Problem Solving.)

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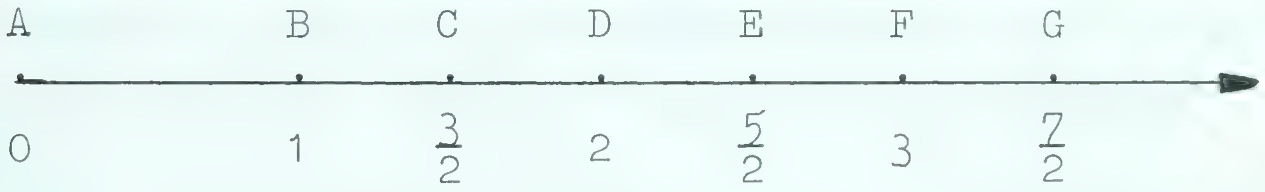
[11] *Ibid*., p. xii.

Measurement

STM Approach. In Unit 8, under the heading "Measure of a Segment" in STM, Book 1, the following lesson is found.

> In unit 1 of this book you compared segments using the ideas of congruence, of "less than," and of "greater than." Now you will compare segments by measuring them. **A** You can use a number line like the one represented in D1 to help you to think about measuring segments. What number is associated with point A? With point B?

[The D1 diagram was not located in this position in the original.]



D1

> **B** $\overline{BD} \cong \overline{AB}$ and $\overline{DF} \cong \overline{AB}$.
>
> What number is associated with point D? With point F?[12]

The lesson proceeds in this manner until the student is led to the idea that the measure of a segment is "The difference of the coördinates of the end-points of a segment."[13]

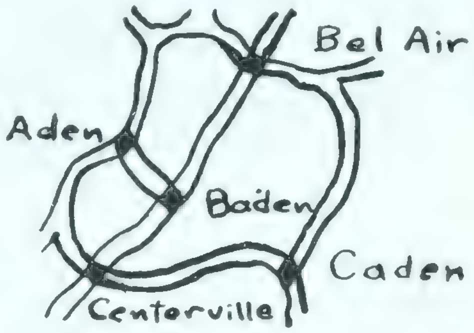
---

[12] Henry Van Engen and others, Seeing Through Mathematics, Book 1, Part 2 (Toronto: W. J. Gage Limited, [n.d.]), p. 427.

[13] Ibid.

EMM Approach. At the beginning of Chapter 7 of EMM, Book 1, the following introduction to measurement is given.

> Here is a map showing several routes from Centerville to Bel Air. Assuming all roads are equally good, if you were in a hurry to go from Centerville to Bel Air, which route would you take? Why? How do you know?



This is a problem of comparing sizes of routes.[14]

Following this introduction, the concept of a one-dimensional figure is introduced and the student is asked to compare one-dimensional figures arranged in pairs. Some can be compared visually, but others require some measuring device like a piece of string. Then, through numerous exploratory exercises, the student is led to recognize the need for a unit of measure or a unit segment. The measure of a segment is developed as the number of arbitrary unit segments that, laid end-to-end, have the same length as the segment being measured.[15]

Winston Mathematics (CON) Approach. Under the heading "Exact and Approximate Numbers" near the beginning of Winston Mathematics, Intermediate 1, the following introduction to measurement is given.

---

[14] Keedy, Jameson, and Johnson, *op. cit.*, p. 287.

[15] *Ibid.*, pp. 288-294.

> The picture shows Old Faithful Geyser which is located in Yellowstone National Park.
>
> Old Faithful erupts, on the average, every 66 minutes. There is always some error in all measurement; hence, it is not possible for a measurement to be exact. The measured time from one eruption to the next is 66 minutes, so we can say that the geyser erupts approximately once in an hour. This is a rough approximation of the measured time between eruptions.
>
> If we count the number of eruptions of the geyser in a day, this number is exact. Numbers which represent measurements are not exact, but numbers which are found by counting are exact.[16]
>
> Exercises follow. The next topic is "How to Round

Off Numbers."

Number Systems

STM Approach. In Unit 7 of STM, Book 1, the following lesson is found.

> In unit 6 you learned that the multiplication of natural numbers is a mapping of the set of ordered pairs of natural numbers onto the set of natural numbers. You also learned that the product of two natural numbers is a natural number. In this lesson you will learn how to find the product of two rational numbers.[17]

A few questions and specific examples follow to review the operation of multiplication with fractions, the

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[16] H. L. Stein and others, Winston Mathematics, Intermediate 1 (Toronto: The John C. Winston Company, Ltd., 1954, 1957), p. 4.

[17] Henry Van Engen and others, Seeing Through Mathematics, Book 1, Part 2 (Toronto: W. J. Gage Limited, [n.d.]), p. 324.

symbols used to represent rational numbers.

> The product of two rational numbers is a rational number indicated by a fraction. The numerator of the fraction is the product of the two numerators, and the denominator is the product of the two denominators. The work that follows will help you use variables to develop the definition of the product of two rational numbers
>
> . . . .
>
> product of two rational numbers. The universe for a and c is N. The universe for b and d is C. For each replacement of a, b, c, and d,
>
> $$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}\ .^{18}$$

EMM Approach. Near the beginning of Chapter 6 in EMM, Book 1, the following treatment of operations with fraction numerals is found.

> Do you remember how to multiply arithmetic numbers, using fraction numerals?[19]

A number of Let's Explore exercises follow in which a pie and an apple are divided into halves and sevenths and used to illustrate the process of multiplication using fraction numerals.

> Do you recall that in order to multiply using fraction numerals you find the product of the numerators and the product of the denominators? This way you find a fraction numeral for your answer. Its numerator is the product of the numerators, and its denominator is the product of the denominators. The reason we do this is that it fits our ideas about the world around us. We might define multiplication in various ways, but to have

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[18] Ibid., pp. 324-325.

[19] Keedy, Jameson, and Johnson, op. cit., p. 237.

it fit our ideas, we define it this way:

> If a and c are any whole numbers and b and d are any natural numbers,
>
> $$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$ [20]

Winston Mathematics (CON) Approach. Under the heading "Important Points in Multiplication of Fractions" the following is found in Book 2.

> A. To multiply a fraction by a whole number or to find a fractional part of a whole number, multiply the numerator of the fraction by the number. Divide this product by the denominator. [Examples are given.]
>
> . . . .
>
> B. To multiply two fractions, multiply the two numerators. Divide this product by the product of the denominators. [An example is given.]
>
> . . . .
>
> C. Sometimes, before we multiply, we may divide the numerator of one fraction and the denominator of another fraction by a number greater than 1 to make the work easier. This use of division is known as cancellation. [An example is given.]
>
> . . . .
>
> D. To multiply mixed numbers, change the mixed numbers to improper fractions and then multiply. [Examples are given.][21]

Numeration

STM Approach. In the STM series the topic of place

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[20] Ibid., pp. 233-234.

[21] H. L. Stein and others, Winston Mathematics, Intermediate 2 (Toronto: The John C. Winston Company, Ltd., 1954, 1957), p. 29.

value in the base-ten numeration system is approached by studying a variety of tally, code, grouping, and place value systems having bases other than ten. Interspersed throughout the discussion of the properties of base-two, base-four, base-seven, and other base numeration systems are parallel examples in the base-ten system. Eventually the treatment leads up to scientific (or exponential) notation as used in the decimal system.[22]

EMM Approach. Following some discussion of Babylonian, Roman, and Hindu numeration systems, the EMM series treats the concept of place value by considering expanded numerals in the decimal system of numeration. Then, exponential notation is treated; and, finally, the properties of the base-five numeration system and other base systems are treated to make the basic patterns in the familiar base-ten system more apparent.[23]

Winston Mathematics (CON) Approach. Under the heading "Picturing Our Number System," the following description is found.

Each of the pockets in the place-value chart

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[22] Henry Van Engen and others, Seeing Through Mathematics, Book 1, Part 1 (Toronto: W. J. Gage Limited, [n.d.]), pp. 206-229.

[23] Keedy, Jameson, and Johnson, op. cit., pp. 1-34.

represents a *place* in which a figure may be written in a number. In 38 the 3 is in the tens' place and the 8 is in the ones' place. Find the pockets which represent these places in the chart.[24]

Geometry

STM Approach.

> *bisector of an angle* . . . A ray determined by the vertex of an angle and a point in the interior of the angle so that the two angles formed by the ray and the sides of the original angle are congruent. If point J is in the interior of $\angle BAC$ and if $\angle BAJ \cong \angle JAC$, then ray AJ is the bisector of $\angle BAC$. [Diagram not reproduced.][25]

EMM Approach.

> To bisect means to cut in half. An angle is bisected by drawing a ray between the sides of the angle so that two smaller angles with the same measure are formed. Here $\angle ABC$ is bisected by $\overrightarrow{BD}$, so $\angle ABD \cong \angle DBC$. The ray $\overrightarrow{BD}$ is called the *angle bisector*. [Diagram not reproduced.][26]

Winston Mathematics (*CON*) *Approach.*

> To bisect an angle by measurement, use a protractor to find the number of degrees in the angle and divide the result by 2. Now mark on your paper a point which

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[24] H. L. Stein and others, *Winston Mathematics, Intermediate 2* (Toronto: The John C. Winston Company, Ltd., 1954, 1957), p. 2.

[25] Henry Van Engen and others, *Seeing Through Mathematics, Book 2, Part 1* (Chicago: Scott, Foresman and Company, 1962), p. 69.

[26] Mervin L. Keedy, Richard E. Jameson, and Patricia L. Johnson, *Exploring Modern Mathematics, Book 2, Teacher's Edition* (New York: Holt, Rinehart and Winston, Inc., 1963), p. 68.

> locates the middle of the angle. Connect this point with the vertex of the angle.[27]

Problem Solving

The principal difference among the texts in their approaches to problem solving is that the STM and EMM texts emphasize student understanding of the structure of the problem and description of the situation in the form of a mathematical sentence, whereas, in the Winston Mathematics series, apparently the important part of problem solving is obtaining the correct answer.

In summary, it might be noted that the STM materials seem rigidly committed to a more precise mathematical presentation than that found in the EMM materials. However, both series do emphasize student discovery and understanding, and both do seem to arrive at comparable generalizations with comparable precision of expression. In contrast, the Winston Mathematics materials seem to place emphasis on manipulative or "how-to-do-it" techniques.

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[27] H. L. Stein and others, Winston Mathematics, Intermediate 2 (Toronto: The John C. Winston Company, Ltd., 1954, 1957), p. 229.

III. DESIGN OF THE SPECIAL MATHEMATICAL UNDERSTANDINGS TEST

Since the primary purpose of the experiment presently being reported was to establish to what extent the three texts involved fulfilled the expectations of the Alberta Junior High School Mathematics Subcommittee's "Guidelines"[28] and since, at the time of the experiment, there were no published mathematics tests available that reflected these expectations, it was decided to build a Special Mathematical Understandings Test, referred to hereinafter as the SMU test. Early in the development of the SMU test, a decision was made to proceed so that the final form of the test would contain fifty tested items dealing with understandings in five of the areas emphasized in the "Guidelines," namely:

1. Measurement,
2. Number Systems,
3. Numeration,
4. Geometry, and
5. Problem Solving.

A set of ten questions in each area was the goal for the final version of the test.

With regard to the other areas mentioned in the "Guidelines," a complete set of questions specifically designed to test understandings of "set concepts," or

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[28] See pp. 4-5 in Chapter I.

"strategies of logic" was not drafted because of the difficulties experienced in separating these understandings from those in the five areas listed above. However, several of the items included in the final form of the test do embody ideas of set and logic in their phrasing and in the understandings involved. Because only one of the texts involved in the experiment deals with "permutations, combinations, probability, statistics, and series," and these topics appear at the end of the grade eight text, items in this area were not included in the development of the test. It was felt that the Iowa Tests of Basic Skills (IOWA) used in the experiment would give sufficient indication of a student's grasp of "conventional mathematics topics and skills," so items in this line were not specifically included in the SMU test.

In designing the items the investigator tried to abstract mathematical understandings from special terminology and symbolism. This approach was felt necessary because the "traditional" students were not acquainted with much of the special terminology and notations often associated with the understandings to be tested and because even the two "modern" texts differ from one another in their use of symbolism. The investigator felt that it was important, if comparisons were to be made, that the SMU test questions should be equally intelligible to all three groups of students involved in the

experiment. He was (and is) also convinced that the understanding of mathematical ideas must take precedence over the manipulation of symbols. The involvement of computational skill was minimized in the SMU test as it was felt that student facility in this regard could be judged from the results of the administration of the IOWA test.

So that the concepts tested would be compatible with those expected of grade eight students, ideas for the SMU test items were gleaned largely from the School Mathematics Study Group (SMSG) grade eight text materials.[29] Ideas for some of the items have their origin in material found in the 26th Yearbook of the National Council of Teachers of Mathematics (NCTM)[30], in a test of arithmetic understandings[31], and in SMSG seventh grade text materials.[32]

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[29] School Mathematics Study Group, Introduction to Secondary School Mathematics, Volume 2 (2 parts) and Introduction to Secondary School Mathematics, Volume 2, Teacher's Commentary (New Haven and London: Yale University Press, 1963); and School Mathematics Study Group, Mathematics for Junior High School, Volume 2 (2 parts) and Mathematics for Junior High School, Volume 2, Teacher's Commentary (New Haven and London: Yale University Press, 1961).

[30] Max A. Sobel and Donovan A. Johnson, "Analysis of Illustrative Test Items," Evaluation in Mathematics, Twenty-sixth Yearbook (Washington: The National Council of Teachers of Mathematics, 1961), pp. 71-91.

[31] Vincent J. Glennon, "A Test of Basic Mathematical Understandings" (Copyright, 1947). (Mimeographed.)

[32] School Mathematics Study Group, Introduction to

During December, 1963 and January, 1964, the investigator carefully drafted items for each of the five areas in which understandings were to be tested (Measurement, Number Systems, Numeration, Geometry, and Problem Solving). He endeavoured to keep the language in the items precise but neutral in the sense that it would be meaningful to any one of the students, regardless of which text had been studied. In early January a mimeographed collection of items entitled *First Draft: Items for Grade Eight Special Test of Mathematical Understandings* was produced. This twenty-one page first draft contained 91 five-alternative, multiple choice questions that the investigator considered to be appropriate for inclusion in the SMU test. Copies of the *First Draft* were circulated to members of the Subcommittee, and to staff members of the Faculty of Education and the Mathematics Department of the University of Alberta. Critical appraisal of the items was invited, and, on the basis of the comments received, many of the items were strengthened by the rephrasing of the stem or alternatives, several inappropriate items were deleted, and a number of new items were added.

Secondary School Mathematics, Volume 1 (2 parts) and *Introduction to Secondary School Mathematics, Volume 1, Teacher's Commentary* (New Haven and London: Yale University Press, 1963); and the School Mathematics Study Group, *Mathematics for Junior High School, Volume 1* (2 parts); and *Mathematics for Junior High School, Volume 1, Teacher's Commentary* (New Haven and London: Yale University Press, 1961).

In the first week of February, a 75-item pilot study version of the test was mimeographed. In this version there were fifteen items from each of the five areas of understanding and the items were arranged so that every fifth item belonged to the same category (Measurement, Number Systems, Numeration, Geometry, or Problem Solving). The test was administered to 138 grade nine students (five classrooms) who were not part of the main experiment. Two of the classes had taken the complete grade eight STM course, two classes had completed _Winston Mathematics, Intermediate 2_, and the fifth class had studied a semi-traditional grade eight mathematics text. The students' answer sheets were scored, and the papers were placed in rank order from that with the highest score to that with the lowest, disregarding instructional group, and the individual student's item responses were recorded. The median score was established and indices of item discrimination were obtained by computing tetrachoric correlations with the criterion dichotomized at the median.[33] Generally speaking, the higher the tetrachoric correlation for an item, the better is its power of discrimination between students who score

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[33] H. J. Casebolt and W. T. Pace, "Chart for Computing Tetrachoric r with the Criterion Dichotomized at the Median" (Raleigh, North Carolina: North Carolina State Personnel Department, Division of Recruitment and Testing, [n.d.]) (Mimeographed.)

well on the test as a whole and those who score poorly. In general, the ten most discriminating items for each of the five areas of understanding were chosen for inclusion in the final form of the test. The difficulty of an item, judged from the percentage of students who answered the item correctly, also had some bearing on the choice of items where two or more had similar powers of discrimination. A copy of the pilot study version of the test and a chart of the item discrimination indices and percentages of students passing each item is included in the Appendix.

The final version of the Special Mathematical Understandings (SMU) Test was mimeographed in late February. The items were arranged so that the first item dealt with a topic from Measurement, the second with Number Systems, the third with Numeration, the fourth with Geometry, the fifth with Problem Solving, the sixth with Measurement, and so on, so that every fifth item belonged to the same category. This arrangement was employed so that, if a student could not complete the test in the allotted time (which was estimated from observations made during the pilot study), no one category of items would receive a disproportionately smaller portion of his attention. Within each category, the items were arranged in the order of increasing difficulty as estimated from the pilot run. A copy of the final version of the SMU test is included in the Appendix.

It is believed that the SMU test has acceptable content validity for the purposes for which it was designed in view of the fact that the concepts that the items purport to test were derived largely from grade eight SMSG materials (the SMSG is a widely recognized and authoritative group in the field of school mathematics). Furthermore, responsible persons in the field of mathematics education have thoroughly studied the items for weaknesses.

IV. THE TESTING PROGRAM

In late September, 1963, the Cooperative School and College Ability Test (SCAT), Form 4A[34] was administered to all students involved in the experiment. Two scores were obtained from the administration of the test--a verbal ability (V) score and a quantitative ability (Q) score. In The Fifth Mental Measurements Yearbook[35], the SCAT test is reported as having a Kuder-Richardson (Formula 20) estimated total reliability score of .95. The test is described in the same source as non-specific and indicative of a student's

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[34] Educational Testing Service, Cooperative School and College Ability Tests, School Ability Test, Form 4A (Princeton: Cooperative Test Division, Educational Testing Service, 1956).

[35] Oscar Krisen Buros (ed.), The Fifth Mental Measurements Yearbook (Highland Park, N.J.: The Gryphon Press, 1959), pp. 453-455.

ability to achieve at school work rather than of his actual achievement level.

In September, 1963, the grade eight portions of the Arithmetic Concepts (AC) and Arithmetic Problem Solving (PS) subtests of the _Iowa Tests of Basic Skills_ (IOWA)[36] were administered to all students involved in the experiment. The IOWA test is reported in the _Fifth Mental Measurements Yearbook_[37] as having a total split-half reliability coefficient of .98 and sub-test reliability coefficients of approximately .80. It is rated as an "evaluative" test designed to measure generalized educational skills over a wide range of ability. According to the reviewers, the IOWA Arithmetic Concepts test very ingeniously measures a wide range of mathematical experiences. "Computation is encountered in meaningful form in the problems subtest, although one gets the impression that the range of topics in this subtest is rather narrow, with an undue emphasis on money at the expense of items on capacity, weight, and measures."[38]

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[36] E. F. Lindquist and A. N. Hieronymus, _Iowa Tests of Basic Skills for Grades 3-9_ (Boston: Houghton Mifflin Company, 1955), pp. 83-86 and pp. 93-95.

[37] Oscar Krisen Buros (ed.), _The Fifth Mental Measurements Yearbook_ (Highland Park, N. J. : The Gryphon Press, 1959), p. 36.

[38] _Ibid_., p. 35.

According to an IOWA test manual[39], the concepts used in building the Arithmetic Concepts (AC) sub-test are associated with knowledge of base-ten and Roman numeration (and coins), whole numbers, fractions, decimals, per cents, standard measures, geometric figures, and ratio and proportion.

The investigator would describe the Problem Solving (PS) sub-test as largely computationally oriented. Many of the questions involve calculations with money in social-type situations.

In early March, 1964, the IOWA AC and PS sub-tests were re-administered. In addition, the SMU test was administered to all participating students. The development of the SMU test is described on pages 75 to 81 of this chapter. Using the responses from the 604 students who wrote the test, a Kuder-Richardson (Formula 20) reliability index of .83 was calculated for the SMU test by following the procedure outlined by Gulliksen.[40] The data used in calculating the reliability index are included in the Appendix.

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[39] Iowa Tests of Basic Skills, Manual for Administrators, Supervisors, and Counselors (Boston: Houghton Mifflin Company, 1956), pp. 69-70.

[40] Harold Gulliksen, Theory of Mental Tests (New York: John Wiley and Sons, Inc., 1950, Third Printing, March 1961), p. 224.

While the testing program was being carried out in March, the teachers involved in the experiment were asked to complete a questionnaire designed to gather information about the characteristics of the teachers and their opinions of the texts they were using. On pages 117 to 127 in Chapter IV a summary of teacher characteristics and opinions gathered from the questionnaire is presented.

V. NULL HYPOTHESES TESTED

The data gathered from the testing program described on the preceding pages were used in statistical analyses designed to test the following null hypotheses:

1. On the SMU test, there are no significant differences
    a) among the group mean scores obtained by STM, EMM, and CON students,
    b) among scholastic ability level mean scores.
2. On the IOWA AC sub-test, there are no significant differences
    a) among the group mean scores obtained by STM, EMM, and CON students,
    b) among scholastic ability level mean scores.
3. On the IOWA PS sub-test, there are no significant differences
    a) among the group mean scores obtained by STM, EMM, and CON students,
    b) among scholastic ability level mean scores.

VI. STATISTICAL PROCEDURES

Each student who had written all of the tests administered in the course of the experiment was assigned an

identification number and his September AC, September PS, V, Q, SMU, March AC, March PS, and SCAT Total scores and his ID number were punched on an IBM card. The data thus entered on the 604 IBM cards were then processed as described in the succeeding paragraphs. A complete listing of the raw scores obtained by each student is included in the Appendix.

Since rigorous experimental controls on the characteristics of the groups could not be employed, the first consideration was to test for significant differences among the groups with respect to mean scholastic ability scores and mean arithmetic achievement scores in September. One-way analyses of variance were carried out to test the hypotheses that:

> There are no significant differences among the mean scores obtained by the groups studying STM, EMM, and CON materials on
> a) the September AC test,
> b) the September PS test,
> c) the September V test,
> d) the September Q test.

The results of the analyses are summarized in Table IV on page 86. (More detail is supplied in the Appendix.)

In Table IV, and in succeeding tables that summarize the results of carrying out an analysis of variance, the following abbreviations are used:

N represents the number of students' scores involved
$\overline{X}$ represents a group mean score

TABLE IV

MEAN SCORES AND F-RATIOS FROM ANALYSES OF VARIANCE ON SCORES FROM THE TESTING CARRIED OUT IN SEPTEMBER, 1963

GROUP	N	$\bar{X}_{AC}$	$\bar{X}_{PS}$	$\bar{X}_{V}$	$\bar{X}_{Q}$
STM	312	23.92	13.84	48.17	38.82
EMM	144	24.20	14.66	47.40	38.79
CON	148	28.12	14.64	48.72	40.80
$F = \frac{MS_{GRP}}{MS_{WTH}}$		14.34**	2.25	1.20	4.47

$F_{.01}(2,601) = 4.61$

$\overline{X}_{AC}$ represents a group mean score on the IOWA Arithmetic Concepts test

MS represents a mean square or variance estimate

MS_{GRP} represents the between-groups mean square

MS_{WTH} represents the within-groups mean square

F represents the F-ratio obtained by dividing MS_{GRP} by MS_{WTH}

$F_{.01}$ (2,601) represents the value of F required for significance at the .01 level with 2 degrees of freedom associated with the MS_{GRP} and 601 degrees of freedom associated with MS_{WTH} (Critical F-ratio).

As shown immediately below Table IV, the critical F-ratio for a .01-level test and the associated degrees of freedom in this instance is 4.61. This means that there is a significant difference between mean scores if the appropriate ratio exceeds 4.61. The use of a .01-level test ensures that a mean score difference would not be considered significant unless it could only occur as a result of sampling error in less than one out of one hundred such sample mean score comparisons.

Inspection of the F-ratios in Table IV shows that there were significant differences among the group mean scores on the September IOWA Arithmetic Concepts (AC) test. The control group obtained the highest mean score on the AC test. However, no significant differences were observed among the group mean scores on the Sequential Tests of Educational Progress (STEP) arithmetic achievement test

administered to the groups in May, 1963. Furthermore, before the September testing, two control group classes that had dropped out were replaced by two others, and two new STM classes were added to the experimental group. Consequently, it was decided to remove the effect of initial differences among the groups with respect to September AC scores by employing statistical controls when testing the hypotheses associated with the March AC scores.

Table IV shows that the results of .01-level tests indicate no significant differences among the group mean scores on the September PS, V and Q tests.

In order to divide the sample groups into scholastic ability levels so that the effects of the different instructional programs on students of different scholastic abilities could be assessed, the frequency distribution of the SCAT Total (V + Q) scores was found and arbitrarily divided into thirds. Consequently, a student with a SCAT Total score of 96 or above was classified as scoring in the upper third of the scholastic ability range. Students with SCAT Total scores from 83 to 95 scored in the middle third of the scholastic ability range, and those students whose SCAT Total scores were 82 or less scored in the low third. Examination of Table I on page 60 will show that students scoring in the upper third of the SCAT Total score distribution for the sample could be considered as having

high scholastic ability (above the 90th percentile on test publisher's norms). Consequently this ability level is referred to hereinafter as the high ability level (HI). Similar considerations led the investigator to describe the students in the middle third of the SCAT Total distribution as being of "high-average" (HAV) ability. Those students who scored in the low third of the SCAT Total distribution are described hereinafter as being average ability (AV) students. Accordingly, each student became classified by experimental group (STM, EMM, or CON) and by scholastic ability level (HI, HAV, or AV). The number of students classified in each cell is indicated in Table V on page 90.

A chi square test carried out on the cell frequencies in Table V indicated that they were not significantly different from being proportional to the row and column totals.[41] Since in this experiment the sample group sizes were in no way related to possible theoretical population sizes, and since the cell frequencies may be considered proportional to the row and column totals, a two-way unweighted means analysis of variance was considered appropriate for testing the null hypotheses associated with the SMU and March PS scores.[42] Similarly, a two-way

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[41] Details of the calculations involved in carrying out the chi square test are given in the Appendix.

[42] B. J. Winer, Statistical Principles in Experimental

TABLE V

CELL FREQUENCIES FOR INSTRUCTIONAL GROUP AND SCHOLASTIC ABILITY LEVEL CLASSIFICATIONS

GROUP	SCAT ABILITY LEVEL			TOTAL
	HI	HAV	AV	
STM	102	98	112	312
EMM	36	56	52	144
CON	55	49	44	148
TOTAL	193	203	208	604

unweighted means analysis of covariance, with September AC as the covariate, was considered appropriate for testing the null hypotheses associated with March AC scores as it was desired to remove the effect of the differences among the groups on the September AC scores.

An assumption underlying the analysis of variance is that the error (within cells) variance is homogeneous. The tenability of this assumption for each analysis of variance carried out was confirmed by employing Bartlett's test as described by Winer.[43] The simpler Hartley test or Cochran test could not be used because of the wide cell frequency range.

Whenever statistical tests on the differences between means were carried out, .01-level critical values were used. Consequently, the conclusions reached in this report on the basis of statistical tests are relatively conservative since the probability of indicating a significant difference between means as a result of sampling error is only .01.

The computational procedures for the two-way unweighted means analysis of variance (ANOVA) described by

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Design (edition II; New York: McGraw-Hill Book Company, Inc., 1962), p. 224; and R. L. Anderson and T. A. Bancroft, Statistical Theory in Research (New York: McGraw-Hill Book Company, Inc., 1952), pp. 278-279.

[43] B. J. Winer, Statistical Principles in Experimental Design (edition II; New York: McGraw-Hill Book Company, Inc., 1962), p. 95.

Winer[44] had been programmed for use with the University of Alberta's computer and the program was made available to the investigator. The ANOVA computer program was used to process the data to carry out statistical tests of null hypotheses 1a), 1b), 3a) and 3b).[45]

Tests on the significance of differences between pairs of group means were carried out as described by Winer.[46] The test statistic is given by

$$t = \frac{\bar{A}_1 - \bar{A}_2}{\sqrt{\frac{2MS_{w \cdot cell}}{n_h q}}}, \qquad df = N - pq, \text{ where}$$

$\bar{A}_1$ is the mean of means for row 1
$\bar{A}_2$ is the mean of means for row 2
$MS_{w \cdot cell}$ is the pooled within cell variance
N is the total number of scores involved
n_h is the harmonic mean of the cell frequencies
p is the number of rows
q is the number of columns
df is the number of degrees of freedom.

The Newman-Keuls procedure[47] was used to make comparisons between ordered means within each of the high (HI), high-average (HAV), and average (AV) ability groupings. In

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[44] _Ibid_., pp. 241-244.

[45] See page 84.

[46] Winer, _op_. _cit_., p. 208.

[47] _Ibid_., pp. 101-103.

this procedure, the harmonic mean of the cell frequencies is used in the calculations and a fairly conservative test results. However, wherever contrasts appeared to be borderline as a result of the application of the Newman-Keuls procedure, it was decided to employ a "t-test"[48] to compare the cell means two at a time so that the different cell frequencies would be taken into account.

Tests of null hypotheses 2a) and 2b)[49] were carried out by extending the two-way unweighted means analysis of variance procedure to a two-way analysis of covariance. The procedure used is an adaptation of that recommended by Winer.[50] This procedure was used because it was desired to partial out the effects on the March AC scores that would be caused by the differences observed among the group mean scores on the September AC test.

The actual computational formulas used by the electronic computer for the analysis of variance and those used by the investigator for the analysis of covariance are presented in the Appendix.

The assumptions underlying the analysis of covariance

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[48] *Ibid*., p. 244.

[49] See page 84.

[50] Winer, *op*. *cit*., p. 605.

were tested as suggested by Winer.[51] With the March AC score as the criterion and with the September AC score as the covariate, the major assumptions underlying the analysis of covariance were found to be satisfied.

The present chapter has been a description of the nature of the components of the experiment, of the experimental design and of the statistical procedures followed in analyzing the data. The following chapter presents the results of the analysis of the data.

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[51] Winer, op. cit., pp. 586-588.

CHAPTER IV

THE RESULTS OF THE INVESTIGATION

I. FINDINGS FROM THE STATISTICAL ANALYSES

In general in this chapter, each of the null hypotheses tested is stated immediately before the presentation of the results of the appropriate statistical tests. A simple interpretation follows the presentation of each set of results.

Analysis of Variance, SMU Test Scores

Null Hypothesis 1

On the SMU test, there are no significant differences

a) among the group mean scores obtained by the STM, EMM, and CON students,
b) among the scholastic ability level mean scores,
c) attributable to interaction.

Table VI lists the SMU test cell means and Table VII summarizes the analysis of variance carried out on the SMU test scores. The abbreviation, SS, used in Table VII for the first time, represents the "sum of squares" or the "sum of the squares of the differences between each score and its particular group mean" [i.e., $\sum_j (X_{ij} - \overline{X}_{i\cdot})^2$].

Conclusions. Since the observed F-ratio (56.98) for comparisons among group mean SMU scores exceeded the

TABLE VI

SMU TEST CELL MEANS

GROUP	HI	HAV	AV	GROUP MEAN
STM	26.55	18.70	14.20	19.82
EMM	28.28	21.16	17.44	22.29
CON	21.07	15.39	11.59	16.02
			GRAND MEAN	19.38

TABLE VII

SUMMARY OF ANALYSIS OF VARIANCE OF SMU TEST SCORES

SOURCE	SS	df	MS	F
GROUP	3,479.19	2	1,739.59	56.98**
ABILITY	10,560.69	2	5,280.35	172.96**
INTERACTION	133.79	4	33.45	1.10
WITHIN	18,166.94	595	30.53	

$F_{.01}(4,595) = 3.32$ $F_{.01}(2,595) = 4.61$

critical value (4.61), Null Hypothesis 1a) was rejected.

Since the observed F-ratio (172.96) for comparisons among ability level mean SMU scores exceeded the critical value (4.61), Null Hypothesis 1b) was rejected.

Since the observed F-ratio (1.10) attributed to interaction did not exceed the critical value (3.32), Null Hypothesis 1c) was accepted.

Interpretations. Significant differences among the group (treatment) SMU test mean scores and among the ability level SMU test mean scores were observed. The interaction effect was not statistically significant.

Since significant differences were found among the three group mean scores on the SMU test, the group means were compared two at a time to discover between which group means significant differences occurred.

Tests on Pairs of Group Mean Scores on the SMU Test

Null Hypothesis 1 a)

On the SMU test, there are no significant differences

(1) between the STM and EMM group mean scores,
(2) between the STM and CON group mean scores,
(3) between the EMM and CON group mean scores.

The results from the appropriate t-tests[1] were as

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[1] Winer, op. cit., p. 208.

follows:

1a) (1): $t_{\text{STM vs EMM}} = 4.18$

1a) (2): $t_{\text{STM vs CON}} = 6.41$ $t_{.01}(595) = 2.33$

1a) (3): $t_{\text{EMM vs CON}} = 10.60$

Conclusions. Since every one of the above t-tests exceeded the critical value (2.33), Null Hypotheses 1a) (1), 1a) (2), and 1a) (3) were all rejected.

Interpretations. The mean scores obtained on the SMU test by the STM and EMM groups were significantly different and the EMM group mean score was the higher.[2] The mean scores obtained on the SMU test by the STM and EMM groups were significantly different from that obtained by the CON group, and the STM and EMM group mean scores were higher.

Tests on Ordered SMU Cell Mean Scores

In order to make comparisons between the groups at each ability level, the Newman-Keuls procedure[3] was used. In Table VIII on page 100, the high ability cell means for the STM, EMM and CON groups are ordered from least to greatest as the table is read from left to right or top to

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[2] See Table VI on page 96.

[3] Winer, *op. cit.*, pp. 101-103.

bottom. For each contrast there is a double entry in the body of the table. The upper entry is the absolute difference between the mean scores being compared. The lower entry, enclosed by parentheses, is the critical .01-level difference between the means. If the absolute difference exceeds the critical difference, the means are considered to be significantly different.

The studentized range statistic, which is used in calculating the critical mean difference, is a function of the number of steps between the means when they are arranged on an ordered scale. It is also a function of the number of degrees of freedom and the rule of behavior used. In the contrasts made in Table VIII, the number of steps between means is either two or three, the number of degrees of freedom is 595, and the rule of behaviour used is specified by setting alpha equal to .01. If the studentized range statistic is denoted by q, the critical difference between means is given by:

$[\sqrt{(MS_{WTH}/n_h)}]q$, where MS_{WTH} is the pooled within-cell variance, and

n_h is the harmonic mean of the cell frequencies.

For the contrasts presented in Table VIII, the following figures are required:

	2 Steps	3 Steps
Studentized Range Statistic, q	3.64	4.12
Critical Mean Difference	2.64	2.99

TABLE VIII

TESTS ON DIFFERENCES BETWEEN ORDERED HIGH ABILITY CELL MEAN SCORES ON THE SMU TEST

GROUP		CON	STM	EMM
	MEANS	21.07	26.55	28.28
CON	21.07	-----	5.48** (2.64)	7.21** (2.99)
STM	26.55		-----	1.73 (2.64)

Referring to Table VIII one finds that each of the STM and EMM high ability cell mean scores on the SMU test was significantly different from the high ability cell CON mean score, and the STM and EMM means were higher. The difference between the high ability cell STM and EMM mean scores on the SMU test was not statistically significant.

Table IX and Table X summarize the findings of the Newman-Keuls procedure as applied to the high-average ability level cell means and to the average ability level cell means on the SMU test.

Referring to Table IX on page 102, one finds that each of the high-average ability level cell means for the STM and EMM groups was significantly different from the high-average ability CON mean score on the SMU test, and the STM and EMM means were the higher. Since the contrast between the high-average ability mean scores on SMU for the STM and EMM groups was discovered to be a borderline case by considering the effects of using the two actual cell frequencies rather than n_h in the calculation of the critical mean difference, a t-test[4] was carried out with the following results:

$$t_{\text{HAV STM vs HAV EMM}} = 2.69 \quad t_{.01}(595) = 2.33$$

One would conclude that the difference between mean SMU

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[4] Winer, op. cit., p. 244.

TABLE IX

TESTS ON DIFFERENCES BETWEEN ORDERED HIGH-AVERAGE ABILITY CELL MEAN SCORES ON SMU

GROUP		CON	STM	EMM
	MEANS	15.39	18.70	21.16
CON	15.39	-----	3.31** (2.64)	5.77** (2.99)
STM	18.70		-----	2.46* (2.64)

TABLE X

TESTS ON DIFFERENCES BETWEEN ORDERED AVERAGE ABILITY CELL MEAN SCORES ON SMU

GROUP		CON	STM	EMM
	MEANS	11.59	14.20	17.44
CON	11.59	-----	2.61* (2.64)	5.85** (2.99)
STM	14.20		-----	3.24** (2.64)

scores obtained by students in the high-average ability range was significant and that the EMM cell mean was the higher.

Reference to Table X shows that in the average ability range, the cell means on the SMU test for the EMM and CON groups were significantly different and the EMM cell mean was the higher. Similarly, the EMM and STM cell means were significantly different with the EMM cell mean the higher. The contrast between STM and CON mean scores on the SMU test in the average ability range was borderline, so a t-test was employed, producing the following results:

$t_{\text{AV STM vs AV CON}} = 2.65 \quad t_{.01}(595) = 2.33$

Consequently, one would assert that in the average ability range there appeared to be a significant difference between STM and CON cell means on SMU with the STM mean the higher.

Summary

On the basis of the preceding tests, the investigator concluded that the EMM students scored significantly higher than the STM students except in the high ability range. At all three ability levels, the STM and EMM students scored higher than the CON students on the SMU test. Figure 1 on the following page graphically illustrates the relationships among the mean scores for the groups on the SMU test. In

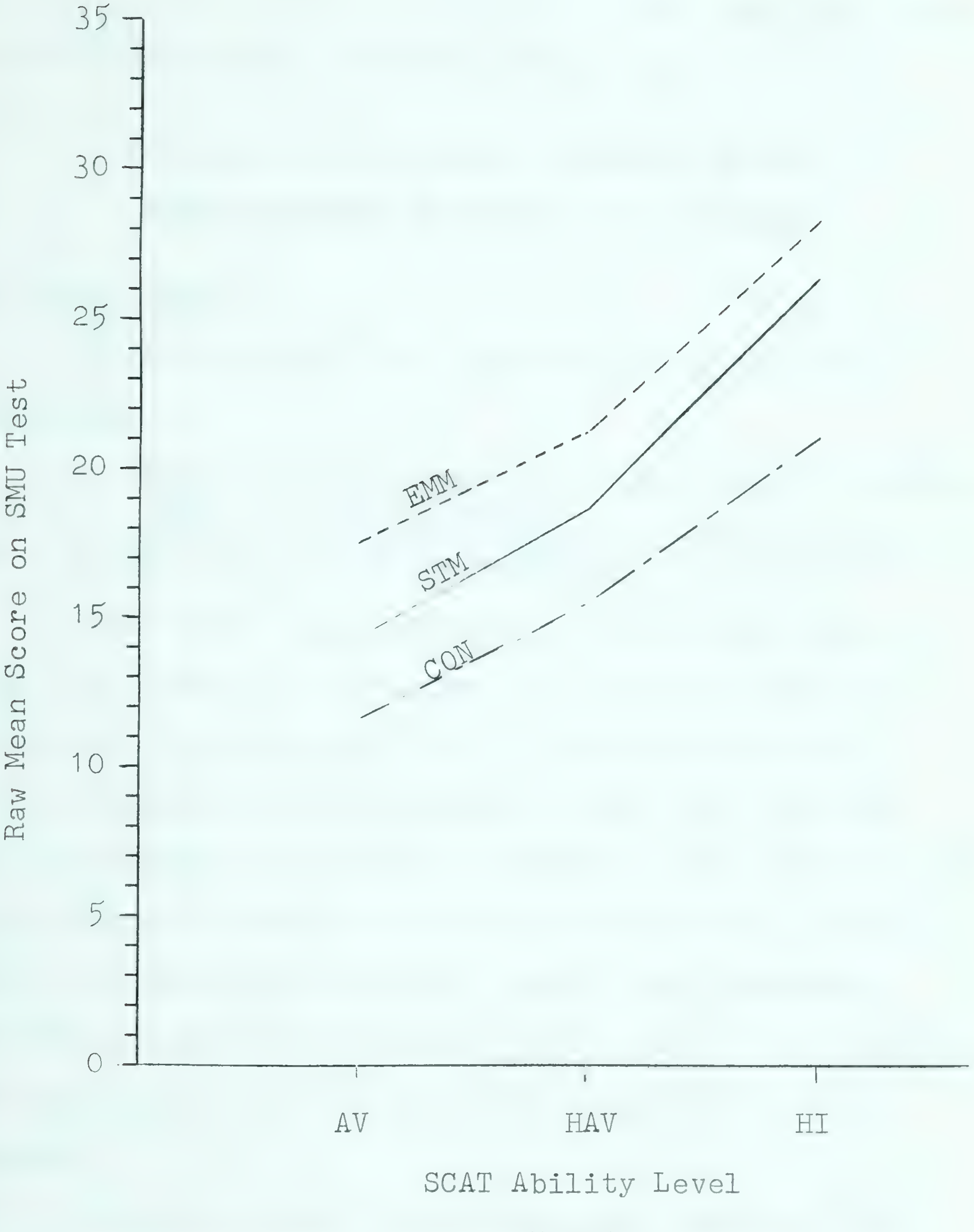

FIGURE 1

THE RELATIONSHIPS BETWEEN MEAN SMU SCORES FOR THE THREE INSTRUCTIONAL GROUPS AT THE THREE ABILITY LEVELS

the Figure the cell mean scores on the SMU test are plotted against SCAT ability level for each group.

Analysis of Covariance, March AC Scores with September AC Scores as Covariates

Null Hypothesis 2

On the March AC test, there are no significant differences

a) among the group mean scores obtained by the STM, EMM, and CON students,
b) among the scholastic ability level mean scores,
c) attributable to interaction.

Since there were significant differences among the group mean scores in September on the AC test and since it was desired to remove this initial bias statistically, Null Hypothesis 2 was tested by carrying out a two-way, unweighted means analysis of covariance. The covariate was the September AC score for each student and the criterion was the corresponding March AC score. The procedures followed and the intermediate results obtained in carrying out the analysis of covariance are presented in the Appendix.

Table XI on page 106 presents the results of the analysis of covariance on the March AC scores with the September AC scores as covariate. Table XII lists the March AC cell means adjusted and unadjusted for the effect of the

TABLE XI

SUMMARY OF ANALYSIS OF COVARIANCE: MARCH AC SCORES ADJUSTED FOR THE EFFECT OF SEPTEMBER AC SCORES

SOURCE	SS'	df	MS'	F'
GROUP	66.01	2	33.01	1.89
ABILITY	1,479.28	2	739.64	42.38**
INTERACTION	76.93	4	19.23	1.10
WITHIN	10,366.06	594	17.45	

$F_{.01}(2,594) = 4.61$ $F_{.01}(4,594) = 3.32$

TABLE XII

MARCH AC CELL MEANS ADJUSTED AND (UNADJUSTED) FOR THE EFFECT OF SEPTEMBER AC SCORES

	HI	HAV	AV	GROUP MEAN
STM	33.44 (37.51)	31.10 (29.67)	27.14 (21.57)	30.56 (29.58)
EMM	34.07 (38.83)	31.21 (30.32)	28.93 (24.02)	31.40 (31.06)
CON	33.00 (38.98)	31.50 (33.26)	28.40 (24.61)	30.97 (32.29)
			GRAND MEAN	30.98 (30.98)

September AC scores.

Conclusions. Examination of Table XII and the critical F-ratios led the investigator to make the following decisions:

Null Hypothesis 2a) was accepted,
Null Hypothesis 2b) was rejected,
Null Hypothesis 2c) was accepted.

Interpretations. The observed adjusted F-ratio (1.89) associated with comparisons among group means did not exceed the critical F-ratio (4.61). Consequently, the differences among the group mean March AC scores adjusted for the effect of September AC scores were not statistically significant. There were significant differences among the adjusted mean scores on the March AC test for the three different scholastic ability levels. The interaction effect was not statistically significant.

Summary. When the effects of the differences among group mean scores on the September AC test were partialled out, there were no significant differences among the group mean scores on the March AC test. Since the contrasts among ability levels were still significant after the initial bias was removed, it might be postulated that the different ability groups learned at different rates

over the course of the 1963-64 experiment, which is as one would expect. On the other hand, the different instructional groups did not show a significant gain or loss with respect to one another in their scores on the AC test. Figure 2 on page 109 displays the relationships among the groups graphically by showing adjusted mean March AC scores plotted against ability level for each instructional group.

Analysis of Variance, March PS Scores

Null Hypothesis 3

On the March PS test, there are no significant differences

a) among the group mean scores obtained by the STM, EMM, and CON students,
b) among the scholastic ability level mean scores,
c) attributable to interaction.

Since no significant differences were observed among the group mean scores on the September PS test, a two-way, unweighted means analysis of variance was carried out with the March PS scores. Table XIII on page 110 lists the cell mean scores on the March PS test, and Table XIV summarizes the results of analysis of variance carried out on the March PS scores.

Conclusions. Examination of Table XIV and the critical F-ratios led the investigator to make the following decisions:

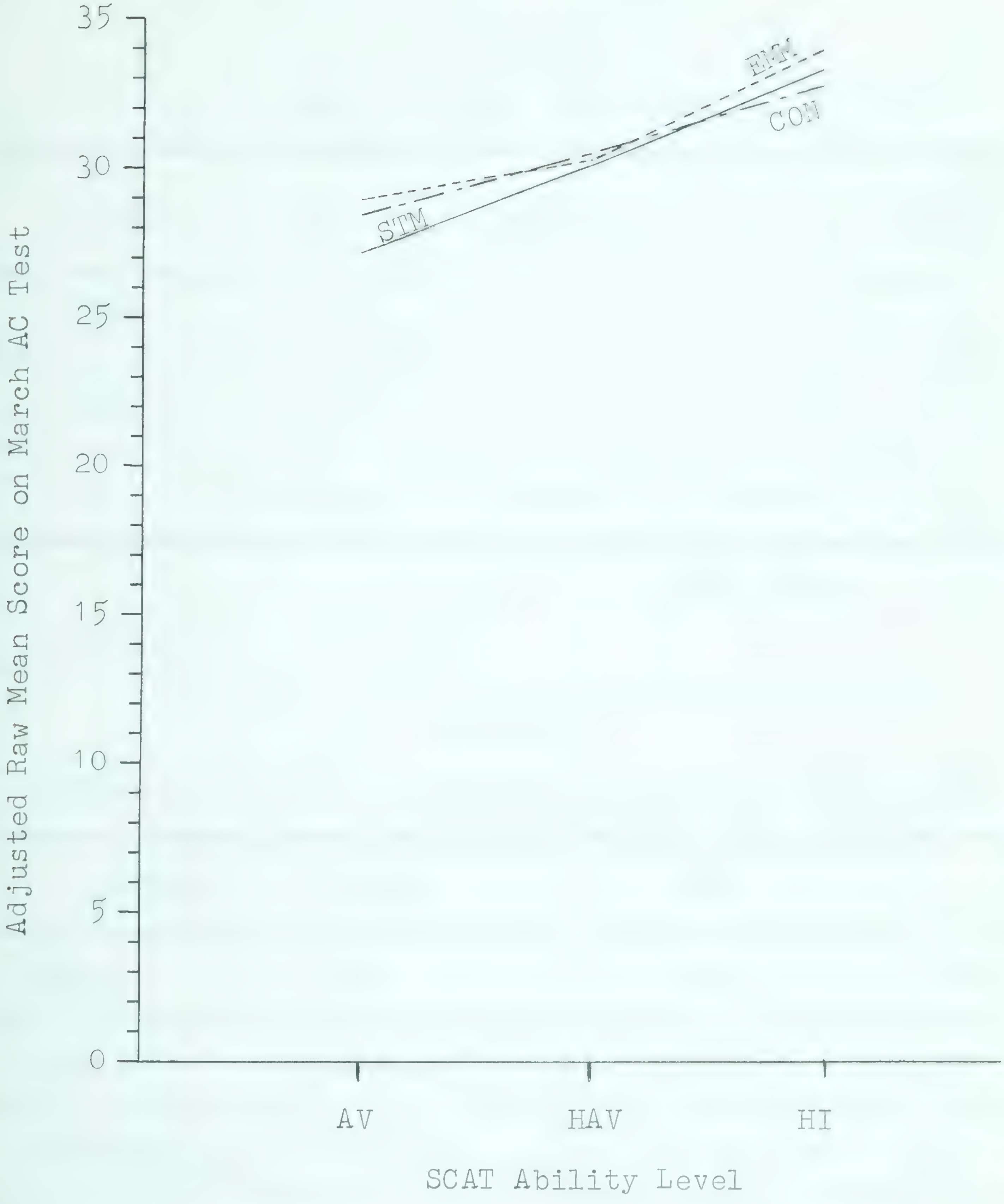

FIGURE 2

THE RELATIONSHIPS BETWEEN MEAN MARCH AC SCORES (ADJ.)
FOR THE THREE INSTRUCTIONAL GROUPS
AT THE THREE ABILITY LEVELS

TABLE XIII

MARCH PS TEST CELL MEANS

GROUP	HI	HAV	AV	GROUP MEAN
STM	20.89	16.23	13.15	16.76
EMM	20.78	16.89	14.17	17.28
CON	22.13	18.12	14.64	18.30
			GRAND MEAN	17.45

TABLE XIV

SUMMARY OF ANALYSIS OF VARIANCE OF MARCH PS TEST SCORES

SOURCE	SS	df	MS	F
GROUP	212.34	2	106.17	5.60**
ABILITY	4,644.98	2	2,322.49	122.52**
INTERACTION	26.11	4	6.53	0.34
WITHIN	11,278.42	595	18.96	

$F_{.01}(2,594) = 4.61$ $F_{.01}(4,594) = 3.32$

Null Hypothesis 3a) was rejected,

Null Hypothesis 3b) was rejected,

Null Hypothesis 3c) was accepted.

Interpretations. There were significant differences among the group (treatment) mean scores and among the ability level mean scores on the March PS test. The interaction effect was not statistically significant.

Tests on Pairs of Group Mean Scores on the March PS Test

Null Hypothesis 3 a)

On the March PS test, there are no significant differences

(1) between the STM and EMM group mean scores,
(2) between the STM and CON group mean scores,
(3) between the EMM and CON group mean scores.

The results of the appropriate t-tests were as follows:

3a) (1) $t_{STM \text{ vs } EMM} = 1.12$

3a) (2) $t_{STM \text{ vs } CON} = 3.29$ $t_{.01}(595) = 2.33$

3a) (3) $t_{EMM \text{ vs } CON} = 2.17$

Conclusions. Null Hypotheses 3a) (1) and 3a) (3) were accepted, but Null Hypothesis 3a) (2) was rejected.

Interpretations. The STM and EMM group means on the

March PS test were not significantly different. The difference between the STM and CON group mean scores on the March PS test was significant and the CON mean score was higher. The EMM and CON group mean scores on the March PS test were not significantly different.

Tests on Ordered March PS Cell Mean Scores

Table XV and Table XVI, which appear on the following page, present summaries of the application of the Newman-Keuls procedure to test differences between March PS cell means in the high ability range and the high-average ability range, respectively.

Examination of Table XV would lead one to conclude that in the high ability range there were no significant differences between the ordered STM, EMM and CON cell means on the March PS test. Table XVI shows that in the high-average ability range there were no significant differences between ordered STM, EMM and CON cell means on the March PS test. However, the CON vs STM contrast was found to be borderline by substituting $n = 112$ for n_h, making the critical mean difference 1.70 (instead of 2.35) The application of a t-test produced the following result:

$t_{\text{HAV CON vs HAV STM}} = 2.58 \quad t_{.01}(595) = 2.33$

Consequently, one could state that there was a significant difference between CON and STM cell means on the March PS

TABLE XV

TESTS ON DIFFERENCES BETWEEN ORDERED HIGH ABILITY CELL MEAN SCORES ON THE MARCH PS TEST

GROUP		EMM	STM	CON
	MEANS	20.78	20.89	22.13
EMM	20.78	-----	0.11	1.35
			(2.08)	(2.35)
STM	20.89		-----	1.24
				(2.08)

	2 Steps	3 Steps
Studentized Range Statistic, q	3.64	4.12
Critical Mean Difference	2.08	2.35

TABLE XVI

TESTS ON DIFFERENCES BETWEEN ORDERED HIGH-AVERAGE ABILITY CELL MEAN SCORES ON THE MARCH PS TEST

GROUP		STM	EMM	CON
	MEANS	16.23	16.89	18.12
STM	16.23	-----	0.66	1.89*
			(2.08)	(2.35)
EMM	16.89		-----	1.23
				(2.08)

test and that the CON cell mean was the higher.

Table XVII on the following page summarizes the findings from the application of the Newman-Keuls procedure to the average ability level cell means, and it shows that in the average ability range there were no significant differences among the STM, EMM or CON cell mean scores on the March PS test.

Summary

On the March PS test, the CON group as a whole scored significantly higher than the STM group but not significantly higher than the EMM group scored. The difference between EMM and STM group mean scores on the March PS test was not statistically significant. In the high and average ability ranges, no significant differences were found between any two cell mean scores on the March PS test. However, in the high-average ability range, a significant difference was found between the CON and STM cell mean scores with the CON mean score the higher of the two. Neither of the EMM vs STM or EMM vs CON contrasts showed significant differences in the high-average ability range. Figure 3 on page 116 displays the relationship among the groups' scores graphically. The cell mean scores on the March PS test are plotted against ability level for each of the three instructional groups.

TABLE XVII

TESTS ON DIFFERENCES BETWEEN ORDERED AVERAGE ABILITY CELL MEANS ON THE MARCH PS TEST

GROUP		STM	EMM	CON
	MEANS	13.15	14.17	14.64
STM	13.15	-----	1.02 (2.08)	1.49 (2.35)
EMM	14.17		-----	0.47 (2.08)

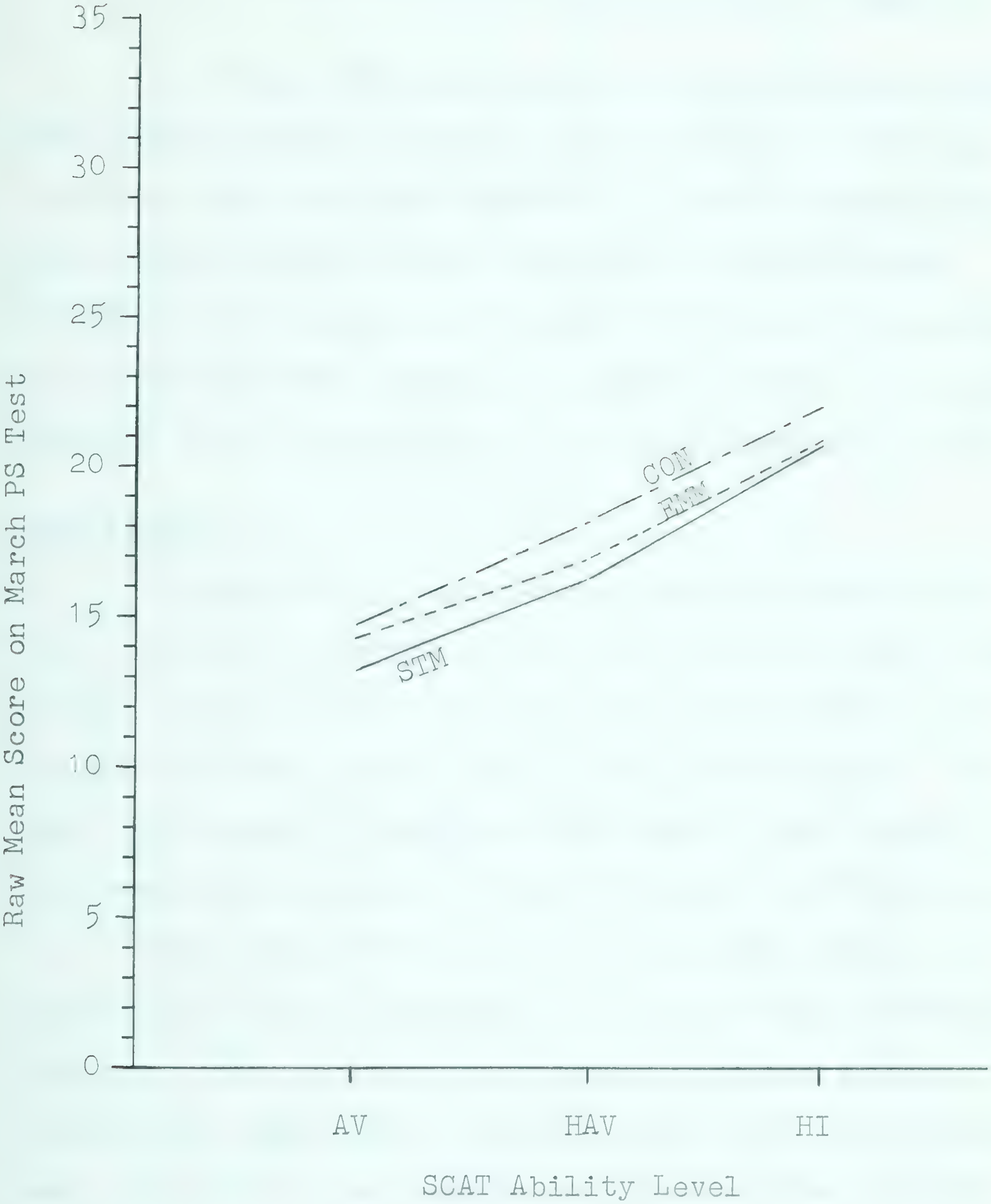

FIGURE 3

THE RELATIONSHIPS BETWEEN MEAN MARCH PS SCORES FOR THE THREE INSTRUCTIONAL GROUPS AT THE THREE ABILITY LEVELS

II. SUMMARY OF TEACHER QUESTIONNAIRE FINDINGS

In March, 1964, the teachers involved in the experiment were asked to complete a questionnaire. The questionnaire had a two-fold purpose: to obtain information on the characteristics of the teachers in the different instructional programs and to solicit teacher evaluations of the STM and EMM programs. A summary of the information gathered from the responses to the questionnaire follows.

Time Factors

Information on the average classroom instruction time received by classes in each of the three instructional groups, on the average teaching load of teachers in each of the instructional groups, and on the average number of years of teaching experience possessed by the teachers in each group is presented in Table XVIII on the following page.

While data of this type does not lend itself to rigorous statistical analysis, it can provide interesting contrasts. For example, even though it appears that, on the average, STM and EMM teachers spent more minutes per week instructing their mathematics classes than the CON teachers did, the STM and EMM teachers were more heavily loaded and had considerably less released instructional time per week than the CON teachers. On the average,

TABLE XVIII

AVERAGE CLASSROOM INSTRUCTIONAL TIME, TEACHING LOAD, AND YEARS OF TEACHING EXPERIENCE FOR EACH GROUP OF TEACHERS

	STM	EMM	CON
Average length of classroom period in minutes	36	38	40
Average time spent in teaching each class involved in the experiment in minutes per week	215	201	180
Average teaching load in minutes per week spent in actual classroom instruction	1348	1380	1200
Average amount of time each teacher was released from classroom instruction or supervision in minutes per week	146	105	204
Average number of years of teaching experience per teacher	10	12	12
Average number of years each teacher had taught grade seven mathematics	5	5	5
Average number of years each teacher had taught grade eight mathematics	6	6	8

the three groups of teachers seemed to have taught similar lengths of time both in total and in the teaching of mathematics at the grades seven and eight levels. On the following page, Figure 4, a bar graph, depicts the proportions of the total grade seven and eight STM or EMM course that each teacher actually covered in approximately sixteen months (1.6 school terms or 80% of a two-term course). The proportions were obtained by dividing the number of chapters actually covered in a particular series by the total number of chapters in the two-year sequence of that series. Consequently, the comparisons are by no means precise, but, considered as a whole, the bar graph does seem to indicate a striking contrast between the degrees to which each group of teachers was able to successfully proceed through a particular course on schedule. The teacher code numbers S1, S2, S3, S4, S5, and S6 represent STM teachers and their classes while the code numbers E1, E2, E3, and E4 represent EMM teachers and their classes. Wherever a teacher instructing more than one class had indicated different coverages for the classes, one bar is shown for each class (For example, bars S1a and S1b represent the coverages for two different classes taught by the same teacher.). The vertical scale on the bar graph represents the proportion of the total course covered. The average course coverage appears to have been between 60 and 65 per cent for the STM

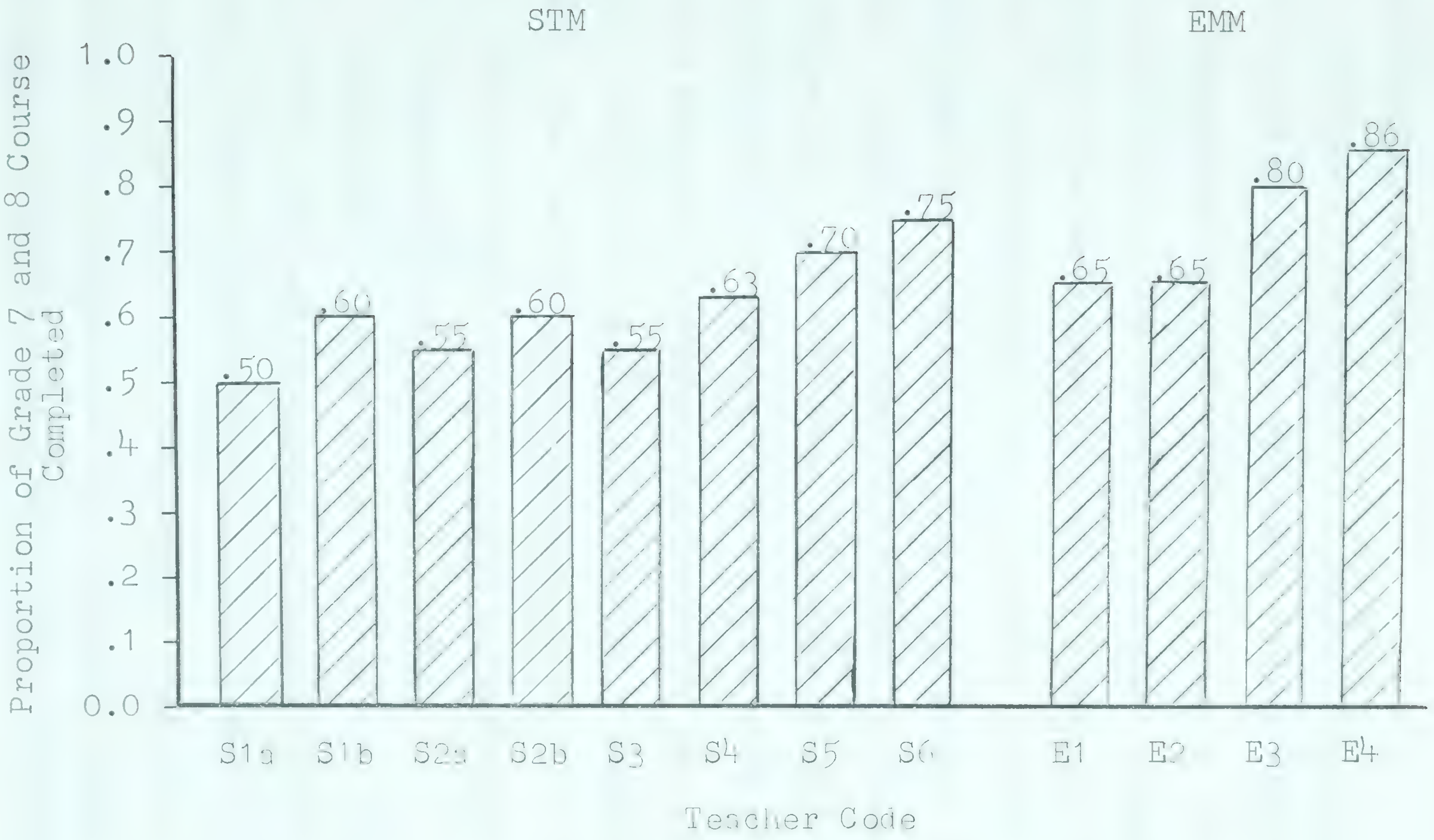

FIGURE 4

PROPORTION OF TOTAL GRADE SEVEN AND EIGHT
STM OR EMM COURSE COMPLETED
BY MARCH 9, 1964

classes and nearly 75 per cent for the EMM classes.

Teacher Characteristics

Although it is virtually impossible to rate a teacher's effectiveness on the basis of his university course background or participation in special courses, or, for that matter, on the basis of the number of years he has taught, it is nevertheless natural to wonder how the mathematics course background and the numbers of years of teaching experience of each of the three groups of teachers compare. Table XIX on page 122 presents in summary form the information collected from the teachers with respect to mathematics course background and with respect to the number of years of teaching experience for each individual teacher. Each of the columns represents a single teacher in one of the instructional programs, and an "X" in a column opposite the description of a course indicates that the teacher represented has taken that course.

Inspection of Table XIX would lead one to conclude that the STM, EMM, and CON groups of teachers had comparable training in their backgrounds to assist them in the interpretation of modern school mathematics curricula. Some of the EMM and CON teachers did have a greater number of conventional mathematics courses in their backgrounds than any of the STM teachers had, but whether this eventuality is significant or not is a matter for conjecture.

TABLE XIX

MATHEMATICS COURSE BACKGROUND AND TEACHING EXPERIENCE OF TEACHERS INVOLVED IN EXPERIMENT

Course Description	STM Teachers						EMM Teachers				CON Teachers					
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Courses that Teachers Considered of Assistance in Interpreting "Modern" School Mathematics																
Algebra (Mathematics 270)	X							X			X		X			
Elementary Mathematics from an Advanced Viewpoint	X							X					X			
Mathematics Curriculum and Instruction	X	X	X				X			X		X				
Inservice "Modern" Mathematics Courses		X			X	X		X	X				X		X	
Monthly Meetings of Teachers of "Modern" Mathematics Programs		X			X											
Other University Math Courses																
Elementary Mathematics	X				X						X					X
Introductory Calculus								X	X	X		X	X		X	X
Calculus and Analytic Geometry								X			X		X			X
Foundations of Mathematics								X	X				X			X
Theory of Finance								X	X				X		X	X
Astronomy								X					X			
Statistics									X							X
Elementary Mechanics															X	X
Analytical Algebra										X		X				
Years of Teaching Experience	2	9	5	6	14	18	3	7	21	8	2	8	7	1	20	11

Thus far, a description of time factors and teacher characteristics has been given. In the following sub-sections a summary of the participating teachers' evaluations of the new text materials is given.

Comments on the Difficulty of the Text Materials

Each point entered in the summaries under the above sub-heading represents one teacher's comments.

Comments from STM Teachers.

1. The theory is too heavy for slower students; the STM materials are too difficult for the grade eight level; the vocabulary is too extensive, but the use of symbols is not; geometry, proofs, and problems in three variables are found to be the most difficult.

2. Some material is too difficult for slow learners--the difficulty lies in a child's ability to read and then reason; the vocabulary and use of symbolism are not too extensive.

3. As with other courses, slow learners find the material difficult even though it is suited to the grade level; the geometry sections are found the most difficult; the vocabulary and use of symbolism are not too extensive.

4. The terms and definitions used need modification for slow students, but the material in the course is suitable even for slow students; the "Precision and Accuracy" section is difficult; the vocabulary and use of symbolism are not found too extensive; the material is challenging.

5. The exercises in Book 2 are found too difficult for slow or mediocre pupils; the text is not easy for a child to read; the vocabulary is "all right"; the text in present format would never do for the average student--the question approach used to develop ideas often obscures generalizations and rules--needs to be condensed and

simplified; the basic ideas in the texts are excellent.

6. The text materials are very difficult; slower students should be segregated for special treatment; the material is suitable for the grade level, but the new terminology is used to no advantage in a burdensome way.

Comments from EMM Teachers.

1. The material is generally not too difficult for students; slow students require more explanation and reinforcement; mathematical proof, negative integers, and generalized definitions give some difficulty.
2. In some cases the material is too difficult for slow pupils; the vocabulary and use of symbolism are suited to the grade level.
3. The section on proofs seems too difficult for any but the best students; there is a large amount of material in the texts that is suitable for even the mediocre student; the section on exponents gives some difficulty; the material is generally suitable for the grade level; no difficulty is encountered with symbols.
4. The "theoretical-axiomatic" approach is too difficult for slow students, but the content is not; many of the proofs seem only of value to brighter students; the material is challenging; no difficulty is encountered with vocabulary or symbols.

Comments on Provisions for Maintaining Computational Skills

Comments from STM Teachers. Four teachers said that the STM program contains sufficient practice and drill for maintaining basic computational skills, but two said that it does not. One teacher indicated that there is not sufficient drill on fractions and another said that there should

be daily drill on basic computational exercises, fundamental operations, integers, fractions, etc.

Comments from EMM Teachers. Three teachers said that the EMM program contains sufficient practice and drill for maintaining basic computational skills, but one teacher indicated that computation is not reviewed often enough.

Comments on the Review of Concepts Presented

Comments from STM Teachers. Four teachers said that the STM program contains sufficient review and reinforcement of the concepts presented, but one teacher said that it does not in every case and that the difficult material "squelches" the interest of many pupils. Another teacher commented that there is sufficient review only if more drill is given when the concepts are introduced.

Comments from EMM Teachers. All four teachers said that there is sufficient review and reinforcement of the concepts presented in the EMM program, but some felt that the teacher must be held largely responsible for supplying supplementary reinforcement.

Comments on Pupil Interest

Comments from STM Teachers. Five teachers said that there was much more student interest in the STM program than

in the currently authorized grade eight mathematics text. However, one teacher said that pupils were not interested because they were under the impression that the series would not be continued the following year. The same teacher indicated that the brighter students were not working.

Comments from EMM Teachers. Three teachers felt that there was greater student interest in the EMM program than in the currently authorized grade eight mathematics text, but one teacher remarked that the materials did not impress some students as being useful, and he indicated that student interest was about the same or a little less than that in the conventional course.

Comments on Problem Comprehension

Comments from STM Teachers. Five of the teachers said that there was improvement in the students' ability to read and understand problems as the course proceeded. One teacher commented that the students used their STM problem solving approach in science (which she also taught). Another teacher said that the students could adjust to a variety of problem situations using the concepts learned.

Comments from EMM Teachers. One teacher felt that there was noticeable improvement in the ability of students to read and understand a problem, and another teacher

mentioned the emphasis on algebraic solutions. However, two teachers felt that there was no noticeable improvement in student problem solving ability. One teacher commented that problem solving is not emphasized in EMM, identifying this as a shortcoming.

III. CHAPTER SUMMARY

In all cases in the treatment of data from this experiment, the difference between two means was considered to be statistically significant only if the probability of observing such a difference as a result of sampling error was .01 or less.

For all comparisons made between group means and between cell means at each of the three ability levels, the EMM mean scores on the SMU test were higher than the STM mean scores which, in turn, were higher than the CON mean scores. The differences between these means were tested for statistical significance by carrying out a two-way analysis of variance, by using appropriate t-tests, and by following the Newman-Keuls procedure. The only difference between means that was not found to be statistically significant was that between the high ability cell means on the SMU test for the STM and EMM groups. All of the other differences between mean scores were found to be significant at the .01 level, favouring EMM over STM and

STM over CON on the SMU test.

A two-way analysis of covariance with the March AC scores as criterion and the September AC scores as covariate showed no significant differences among the STM, EMM, and CON group means.

For all comparisons made between group means and between cell means at each of the three ability levels, the CON mean scores on the March PS test were higher than the EMM mean scores which, in turn, were higher than the STM mean scores. The differences between these means were tested for statistical significance by carrying out a two-way analysis of variance, by using appropriate t-tests, and by following the Newman-Keuls procedure. The difference between the CON group mean and the EMM group mean on the March PS test was not statistically significant, nor was the difference between the EMM group mean and the STM group mean. However, the CON group mean on the March PS test was significantly different from the STM group mean. The only other difference between mean scores that was found to be statistically significant was that between the high-average ability level cell means for the CON and STM groups on the March PS test. Both differences favoured the CON group over the STM group. All other cell mean contrasts within ability levels did not result in significant differences at the .01 level.

Examination of teacher responses to the questionnaire led the investigator to the following conclusions:

1. The teaching groups involved in the experiment were not markedly different from one another with respect to classroom time spent in teaching the classes involved in the experiment or with respect to teaching experience. The CON teachers did indicate an average of one and one-half hours per day more released teaching time than the STM and EMM teachers did.

2. On the average, the EMM teachers had covered a greater proportion of their course than the STM teachers had (75 per cent versus about 63 per cent) by the time 80 per cent of the two-year course time had elapsed.

3. The STM, EMM, and CON teachers, considered as groups, evidenced similar amounts of special training in their backgrounds that would assist them in the interpretation of "modern" school mathematics curricula. Some of the EMM and CON teachers did have more extensive conventional mathematics course backgrounds than the STM teachers had, but the effect that this background would have on their ability to interpret "modern" mathematics curricula is an open question.

4. Some teachers found the theory, some exercises, and some of the terminology in STM to be too difficult for slower students. The EMM terminology was not found difficult, but many students did have trouble with mathematical proof.

5. The majority of the teachers felt that computational skills were maintained effectively in the STM and EMM materials, but a few did not.

6. Both the STM and EMM programs received generally favourable comments with respect to review of concepts presented and with respect to the arousal of pupil interest.

7. The STM approach to problem solving was praised by the STM teachers. Some concern was expressed regarding the lack of a unified problem solving approach in the EMM materials.

CHAPTER V

SUMMARY, CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS FOR FURTHER RESEARCH

I. PURPOSE OF THE STUDY

The purpose of the study was to gather experimental evidence that could be used as a basis for evaluating the relative effectiveness of "modern" and "traditional" grade eight mathematics text materials. The experiment was so designed that data could be gathered and analyzed to shed light on the following questions:

1. Do grade eight students score any higher on a special test of basic mathematical understandings if they study Seeing Through Mathematics or Exploring Modern Mathematics text materials rather than the Winston Mathematics text?
2. Do grade eight students score as well on a conventional arithmetic test if they study Seeing Through Mathematics or Exploring Modern Mathematics text materials rather than the Winston Mathematics text?
3. How do grade eight students who have studied Seeing Through Mathematics compare with grade eight students who have studied Exploring Modern Mathematics on the basis of scores obtained on a special test of basic mathematical understandings and on the basis of scores obtained on a conventional arithmetic test?
4. If the use of Seeing Through Mathematics or Exploring Modern Mathematics texts does have a noticeable effect on the mathematical performance of grade eight mathematics students in general, does this effect hold for students at various levels of scholastic ability or is the effect any more pronounced at one ability

level than it is at another?

5. How are the Seeing Through Mathematics and Exploring Modern Mathematics series regarded by teachers who have used them?

II. THE NATURE OF THE EXPERIMENT

Twenty-two classes of grade eight students and fourteen teachers were involved in the experiment in the 1963-64 school term. Eleven classes studied Seeing Through Mathematics (STM) materials, five classes studied Exploring Modern Mathematics (EMM) materials, and six classes studied Winston Mathematics (CON) text materials.

Late in September, 1963, the Cooperative School and College Ability Test (SCAT), Level Four, and the Iowa Tests of Basic Skills (IOWA), Form 1 (Arithmetic Concepts and Problem Solving Sections) were administered to all students involved in the experiment. In March, 1964, the IOWA tests were readministered along with a Special Mathematical Understandings Test (SMU) that had been developed by the investigator.

On the basis of SCAT Total scores obtained by the students, average (AV), high-average (HAV), and high (HI) scholastic ability categories were established. As a result, each student was identified as belonging to a particular instructional group (STM, EMM, or CON) and a particular scholastic ability level (AV, HAV, or HI).

Null hypotheses[1] were drafted from the questions[2] for which the study was designed. The acceptability of each of the null hypotheses associated with the IOWA Problem Solving and the Special Mathematical Understandings test scores was tested by carrying out two-way analyses of variance, by employing appropriate t-tests, and by using the Newman-Keuls procedure on scores obtained by the students on those two tests. The acceptability of each of the null hypotheses associated with the IOWA Arithmetic Concepts test scores was tested by carrying out a two-way analysis of covariance on the March Arithmetic Concepts scores, using the September Arithmetic Concepts scores as covariate.

In addition to the statistical evidence and by means of a teacher questionnaire, evaluations of the Seeing Through Mathematics and Exploring Modern Mathematics series were obtained from teachers involved in the experiment. The responses to the questionnaire also supplied information about the rate of progress of the classes involved in the experiment, about teaching time factors, and about teacher characteristics.

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[1] Chapter I., p. 7.

[2] Chapter I, bottom of p. 5.

III. A SUMMARY OF THE RESULTS OF THE INVESTIGATION

In the statistical analyses carried out on the data gathered in this experiment, the difference between two mean scores was considered to be statistically significant only if the probability of observing such a difference as a result of sampling error was .01 or less. It was found that students who had studied *Exploring Modern Mathematics* in this experiment obtained a group mean score on the *Special Mathematical Understandings* test that was significantly higher than that obtained by the students who had studied *Seeing Through Mathematics* or *Winston Mathematics*. Not only was this true for comparisons among group mean scores but also for mean score comparisons at every scholastic ability level except the highest, in which case the mean score obtained by EMM students was higher than that obtained by the STM students, but the difference was not statistically significant. However, the *Seeing Through Mathematics* students, as a group and at each scholastic ability level, *did* score significantly higher on the *Special Mathematical Understandings* test than the *Winston Mathematics* students.

The students' March scores on the *IOWA Arithmetic Concepts* test were adjusted for the effects of their September scores on the same test for reasons given on

pages 87 and 88. It was then found that there were no significant differences among the IOWA Arithmetic Concepts test mean scores obtained by the groups of students that studied Seeing Through Mathematics, that studied Exploring Modern Mathematics, and that studied Winston Mathematics.

Analysis of the March IOWA Problem Solving test scores showed no significant differences between group means or ability level means when the Seeing Through Mathematics students' scores were compared with the Exploring Modern Mathematics students' scores or when the Exploring Modern Mathematics students' scores were compared with the Winston Mathematics students' scores. However, the Winston Mathematics students' group mean score on the March IOWA Problem Solving test was significantly higher than the Seeing Through Mathematics students' group mean score. The Winston Mathematics students' high-average ability level mean score on the March PS test was significantly higher than that of the Seeing Through Mathematics students, but the STM versus CON mean score contrasts for the high ability and average ability levels did not show significant differences.

Examination of teacher responses to the questionnaire led the investigator to the following conclusions:

1. The teaching groups involved in the experiment were not markedly different from one another with respect to classroom time spent in

teaching the classes involved in the experiment or with respect to teaching experience. The Winston Mathematics teachers did indicate an average of one and one-half hours per week more released teaching time than the Seeing Through Mathematics and Exploring Modern Mathematics teachers did.

2. On the average, the Exploring Modern Mathematics teachers had covered a greater proportion of their course than the Seeing Through Mathematics teachers had (75 per cent versus about 63 per cent) by the time 80 per cent of the two-year course time had elapsed.
3. The Seeing Through Mathematics, Exploring Modern Mathematics, and Winston Mathematics teachers, considered as groups, evidenced similar amounts of special training in their backgrounds that would assist them in the interpretation of "modern" school mathematics curricula. Some of the Exploring Modern Mathematics and Winston Mathematics teachers did have more extensive conventional mathematics course backgrounds than the Seeing Through Mathematics teachers had, but the effect that this background would have on their ability to interpret "modern" mathematics curricula is an open question.
4. Some teachers found the theory, some exercises, and some of the terminology in Seeing Through Mathematics to be too difficult for slower students. The Exploring Modern Mathematics terminology was not found difficult, but many students did have trouble with mathematical proof.
5. The majority of the teachers felt that computational skills were maintained effectively in the Seeing Through Mathematics and Exploring Modern Mathematics materials, but a few did not.
6. Both the Seeing Through Mathematics and Exploring Modern Mathematics programs received generally favourable comments with respect to the review of concepts presented and with respect to the arousal of pupil interest.
7. The Seeing Through Mathematics approach to problem solving was praised by the Seeing Through Mathematics teachers. Some concern was expressed regarding the lack of a unified problem solving approach in the Exploring Modern Mathematics materials.

IV. CONCLUSIONS

On the basis of the results of the statistical analyses carried out and in consideration of teacher responses to the questionnaire, the answers to the questions posed on pages 130 and 131 can be framed as follows:

1. The *Seeing Through Mathematics* students and the *Exploring Modern Mathematics* students scored significantly higher on a special test of basic mathematical understandings than the *Winston Mathematics* students.
2. As a group, the *Exploring Modern Mathematics* students scored as well on both parts of a conventional arithmetic test as the *Winston Mathematics* students. As a group, the *Seeing Through Mathematics* students scored as well as the *Winston Mathematics* students on the *Arithmetic Concepts* subtest of the conventional arithmetic test but significantly lower than the *Winston Mathematics* students on the *Problem Solving* subtest of the conventional arithmetic test.
3. The *Exploring Modern Mathematics* students scored significantly higher as a group than the *Seeing Through Mathematics* students did on a special test of basic mathematical understandings. The *Exploring Modern Mathematics* students and the *Seeing Through Mathematics* students scored equally well on a conventional arithmetic test.
4. While students at all three ability levels scored significantly higher on the special test of basic mathematical understandings if they studied the *Seeing Through Mathematics* materials or the *Exploring Modern Mathematics* materials rather than the *Winston Mathematics* text, students in the highest ability range who studied STM and EMM texts appeared to receive the most benefit. At the highest ability level the difference between the mean score obtained on the mathematical understandings test by the *Seeing Through Mathematics* students and that obtained by the *Exploring Modern Mathematics* students was not significant. However, at the two lower ability levels, the differences

between the mean scores obtained on the mathematical understandings test by STM students and those obtained by EMM students were significant, favouring the Exploring Modern Mathematics program.

5. Responses from the teachers involved in the experiment indicated unanimous approval of the content and spirit of the Seeing Through Mathematics and Exploring Modern Mathematics textbook series. In spite of some dissatisfaction with textbook format and with the approach to certain topics, there seemed to be universal agreement that these new texts are more satisfactory than the old.

Assuming that the Special Mathematical Understandings test is a valid measure of the kinds of mathematical understandings one would expect from grade eight students, it would appear that the Exploring Modern Mathematics program facilitated more effective teaching of mathematical understandings than either the Seeing Through Mathematics program or the Winston Mathematics text. Under the same assumption, the Seeing Through Mathematics program facilitated more effective teaching of mathematical understandings than the Winston Mathematics text. Assuming the validity of the conventional arithmetic test, the Iowa Tests of Basic Skills, Form 1 (Grade Eight Arithmetic Concepts and Problem Solving subtests), one would conclude that the Exploring Modern Mathematics and Winston Mathematics texts were equally effective in developing the students' traditional mathematics skills. The Seeing Through Mathematics text would be regarded as somewhat less

effective in this respect. Therefore, on the basis of student performance on the tests used in this study one would very likely choose the Exploring Modern Mathematics program rather than either the Seeing Through Mathematics or Winston Mathematics texts. However, on the basis of teacher evaluations of the series, the choice between Seeing Through Mathematics and Exploring Modern Mathematics is not so clear cut. In fact, judging from teacher evaluations, the Seeing Through Mathematics series has a more unified approach to problem solving than that in the Exploring Modern Mathematics series, and this might be considered a strong point in favour of STM.

If it is accepted that teachers are enthusiastic about the problem solving approach in Seeing Through Mathematics, then it is certainly contrary to expectations that the Seeing Through Mathematics students scored significantly lower than the Winston Mathematics students on the IOWA Problem Solving test. However, if one considers the content of the IOWA Problem Solving test items, it is found that, by and large, they tend to measure computational skills rather than problem analysis. Therefore, it does not necessarily follow from the analysis of the test results that the STM students were poorer problem solvers than the Winston Mathematics students. On the other hand, one might suspect that perhaps Seeing Through Mathematics students might not

develop as much facility in computation as Winston Mathematics students would.

From consideration of mean scores on the Special Mathematical Understandings test and on the Iowa Tests of Basic Skills for each ability level, it appears that the Seeing Through Mathematics and Exploring Modern Mathematics series are appropriate for students of low scholastic ability as well as those of high scholastic ability. Furthermore, assuming the validity of the Special Mathematical Understandings test, one might hypothesize that the Exploring Modern Mathematics materials are better suited than the Seeing Through Mathematics materials for helping teachers to develop the mathematical understandings of average and below average students.

While the foregoing conclusions do seem to follow from consideration of the results of the statistical tests, perhaps the interpretations of the results should be tempered by noting that the absolute differences between the group mean scores obtained by the Seeing Through Mathematics students and by the Exploring Modern Mathematics students on the SMU test were only two to three marks (out of a possible total of fifty). Similarly, the absolute difference between the group mean scores obtained by Seeing Through Mathematics students and by Winston Mathematics students on the IOWA Problem Solving test is

less than two marks (out of a total possible of thirty-four). Perhaps the safest conclusion that can be drawn from the study is that certainly the Seeing Through Mathematics and Exploring Modern Mathematics programs offer students something more in the development of mathematical understandings than the Winston Mathematics text does and that, in exchange for this gain, little, if anything, is lost in the development of conventional arithmetic skills.

As noted in the summary of Chapter II, studies similar to the one presently being reported have led to the following conclusions:

1. In general, students studying "modern" mathematics materials and students studying traditional mathematics materials perform equally well on conventional mathematics tests.
2. Whenever a test reflecting the spirit and objectives of "modern" mathematics curricula is included in a study, the "modern" group scores much higher than the traditional group.
3. The new curricular materials appear to be appropriate for all levels of scholastic ability.

It has been found in the present study that such conclusions would seem warranted when applied to the Seeing Through Mathematics and Exploring Modern Mathematics programs in comparison with the Winston Mathematics program. Furthermore, the results of the study tend to favour the Exploring Modern Mathematics materials over the Seeing Through Mathematics materials. The investigator believes that this may be due, at least in part, to the somewhat more

relaxed, less formal presentation of the material in the Exploring Modern Mathematics text. Frankly, both series are enjoyable to read, but the investigator would maintain that the Exploring Modern Mathematics series might be more effective in developing student mathematical understanding than the Seeing Through Mathematics series because the former seems less encumbered with symbolic representations.

V. LIMITATIONS

The investigator freely admits that there are some fairly obvious limitations that must be placed on the interpretation of the findings from this study. First of all, the sample used was not a random sampling from the Alberta student population. However, reference to Table I on page 60 shows that a wide range of student scholastic ability levels was represented in the sample. As another instance of limitation, one might question the advisability of removing the effects of September IOWA Arithmetic Concepts test score differences between the groups when making statistical comparisons on the basis of March IOWA Arithmetic Concepts test scores. However, in view of the changes in the composition of the sample from grade seven to grade eight, it is felt that this was the only course left open to the investigator. A third limitation might well lie in the construction of the Special Mathematical

Understandings test. Since this instrument is such a critical factor in assessing the findings of the study, the investigator strongly urges the reader to carefully scrutinize the items in the final version of the Special Mathematical Understandings test included in the Appendix to see if they do appear to test mathematical understandings that he would judge to be important at the grade eight level. Are there other areas of understanding that could or should be tested? One should also look carefully at the IOWA test, especially the Problem Solving sub-test to see what is or is not tested.

VI. IMPLICATIONS FOR FURTHER RESEARCH

Perhaps the most apparent implication for future research is that the experiment reported herein should be replicated (perhaps at another grade level) with tighter experimental controls. Procedures might well be employed to ensure random sampling in establishing the groups to be used in the experiment. It would be advantageous to have a test, re-test situation for a test like the Special Mathematical Understandings test as well as for the other tests used.

Comments from the teachers involved in the experiment leads one to speculate as to what effects the presence or absence of a unified problem-solving approach might have on

a student's ability to solve problems. An interesting by-product of the analyses carried out on the experimental data was the finding that 30 per cent of the Winston Mathematics students answered a simple "ratio" problem on the Special Mathematical Understandings test while only 10 to 13 per cent of the Seeing Through Mathematics or Exploring Modern Mathematics students answered it correctly. Since at least one of these "modern" programs places heavy emphasis on ratios in problem solving, this phenomenon should certainly be investigated further.

While a study such as the one described in this report serves the purpose of providing overall comparisons between programs, there is a need for studies that concentrate on specific aspects of the various programs being compared. For instance, a detailed study of student facility in solving mathematics problems would do much to explain the phenomenon mentioned in the preceding paragraph as well as to point the way to improved techniques in the teaching of problem solving. There is a need for investigations of the effectiveness of teaching for discovery. Does emphasis on broad understandings and generalizations in the teaching of mathematics detract from student facility with basic arithmetic skills? There are many such unanswered questions that deserve detailed study.

It is hoped that, as more studies in the area of

mathematics curriculum evaluation are conducted at the various grade levels, the information from such studies will become generally available to those who must make decisions about the adoption in a school system of one or more of the many commercial texts that will surely become available in the next few years.

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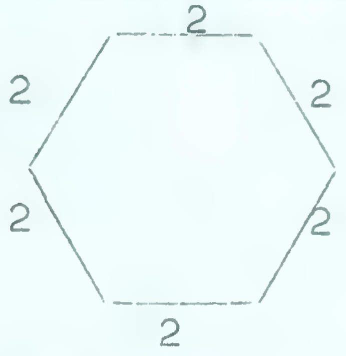
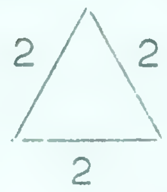
APPENDIX

PILOT STUDY VERSION
OF THE SMU TEST

1. How many of the triangular units of area in the figure to the right below would be needed to cover the area enclosed by the hexagon?





(a) 6
(b) $5\frac{1}{2}$
(c) 3
(d) 4
(e) Cannot be determined from the data given.

2. Which of the following cannot be expressed as a natural number (counting number or zero)?

(a) $\frac{8}{4}$
(b) $\frac{64}{3}$
(c) $\frac{49}{7}$
(d) $\frac{32}{2}$
(e) $\frac{3}{3}$

3. Which of the following is not another name for 5?

(a) V
(b) 𝍸
(c) (11 - 6)
(d) $\frac{1}{3}(15)$
(e) $5^2 - 5$

4. If the measure of one angle of a scalene triangle is 40 degrees, which of the following statements is always true?

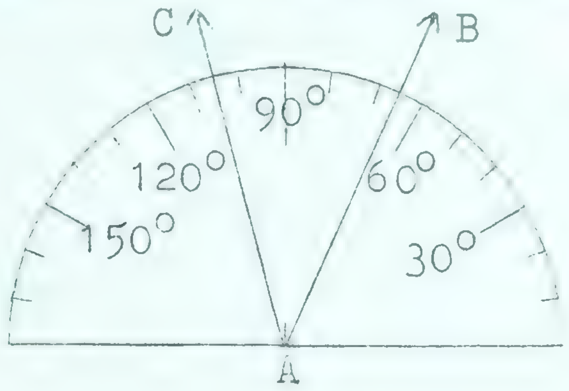
(a) One of the other angles has a measure of 90 degrees.
(b) One of the other angles has a measure of 40 degrees.
(c) The sum of the measures of the other two angles is 140 degrees.
(d) Two of the sides have equal measures.
(e) One of the other angles has a measure of 140 degrees.

5. Which of the following is the correct number phrase for: "Multiply the sum of seven and three by five."

(a) $5 \times 7 + 3$
(b) $5 \times (7 + 3)$
(c) $7 \times 3 \times 5$
(d) $7 + 3 \times 5$
(e) $5(7) + 3(7)$

6. The measure of angle BAC is approximately

(a) 65°
(b) 105°
(c) 40°
(d) 50°
(e) 30°



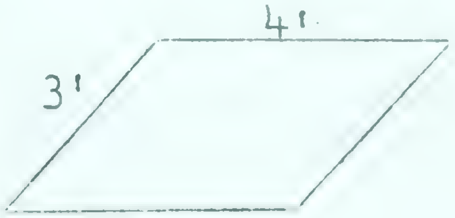
7. In which of the following is 36 completely factored (expressed as the product of primes)?

(a) 4 X 9
(b) 2 X 3 X 6
(c) 3 X 12
(d) 2 X (9 + 9)
(e) 2 X 2 X 3 X 3

8. 3.14 X 10^2 is how many times as large as 3.14?

(a) one hundred
(b) two
(c) ten
(d) one
(e) one thousand

9. The area of the parallelogram shown at the right below



(a) can be found by adding 3 and 4
(b) can be found by multiplying 3 by 4
(c) can be found by multiplying 3 by 4 and dividing the product by 2
(d) can be found by multiplying (3 + 4) by $\frac{1}{2}$
(e) cannot be found from the information given

0. A father is nine times as old as his son. If the son is now "n" years old, how old will the father be in 8 years?

(a) n + 8 years
(b) 9(n + 8) years
(c) 9n + 8 years
(d) n + 9(8) years
(e) 9n years

11. If the measurement of the length of a line segment is recorded as $2\frac{3}{8}$ inches, it is assumed that the true measure of the length of the line segment

(a) is $2\frac{3}{8}$ inches

(b) could be as small as $2\frac{2}{8}$ inches or as large as $2\frac{4}{8}$ inches

(c) could be as small as $2\frac{1}{8}$ inches or as large as $2\frac{5}{8}$ inches

(d) could be as small as $2\frac{5}{16}$ inches or as large as $2\frac{7}{16}$ inches

(e) could be as small as $2\frac{3}{16}$ inches or as large as $2\frac{9}{16}$ inches

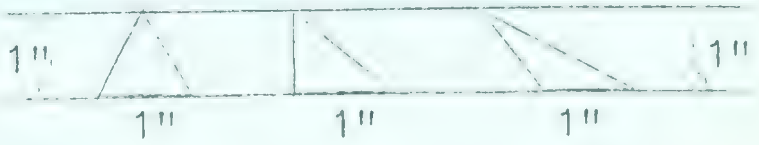
12. If a, b and c represent numbers, which of the following is a representation of the idea that "the order in which numbers are multiplied together has no effect on the product"?

(a) $a \times b = a \times b$
(b) $a(b + c) = a(c + b)$
(c) $a + (bc) = (bc) + a$
(d) $a \times b = b \times a$
(e) $a(bc) = (ab)c$

13. If N represents any natural number (counting number or zero), which of the following will always represent an odd number?

(a) N
(b) $N + 1$
(c) $N + 2$
(d) $2N + 2$
(e) $2N + 1$

14. What can be said about the triangles to the right below?



(a) They are all equilateral.
(b) They are all isosceles.
(c) They each enclose equal areas.
(d) They are identical to one another.
(e) Nothing in particular can be said about the triangles.

15. If 8 + c is less than 12, then

(a) c could be any number less than 4
(b) c could be any number greater than 4
(c) c = 4
(d) c could be any number except 4
(e) c could be any number

16. The greatest possible error in the sum of measurements of 5.5 inches, 4.4 inches and 3.2 inches would be

(a) 0.15 inches
(b) 0.3 inches
(c) 0.1 inches
(d) 3.2 inches
(e) 0.05 inches

17. If a, b and c represent numbers, which of the following is not correct?

(a) $ab = ba$
(b) $a(b + c) = ab + ac$
(c) $a + (b + c) = (a + b) + c$
(d) $a + (b \times c) = (a + b) \times (a + c)$
(e) $a + b = b + a$

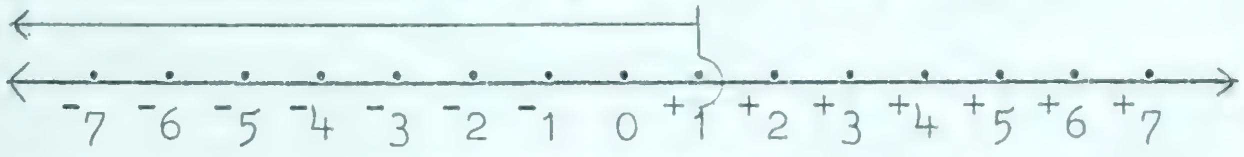
18. Which of the numerals below represents an odd number?

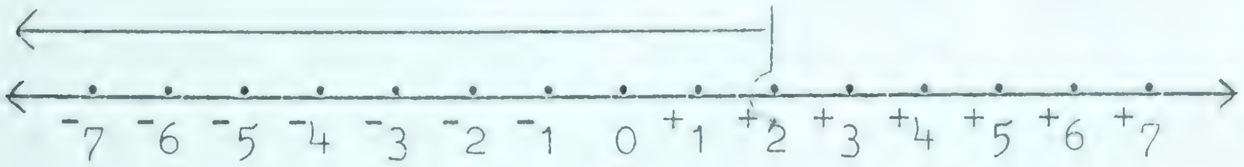
(a) 52,100
(b) 7,390
(c) 1,437
(d) 9,000
(e) 7,564

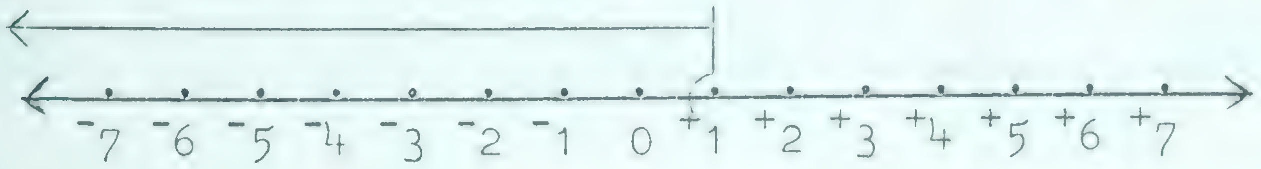
9. If one angle of an isosceles triangle measures 66 degrees,

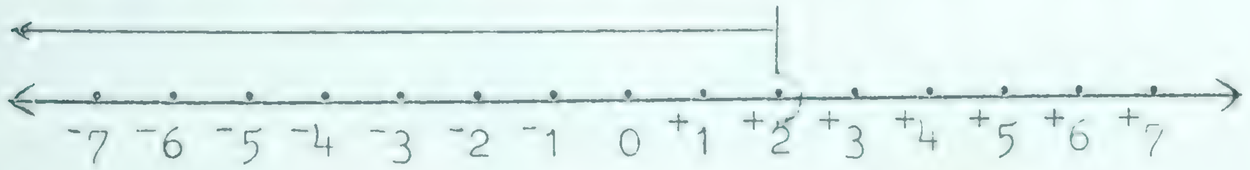
(a) one of the other two angles must measure 66 degrees
(b) one of the other two angles must measure 48 degrees
(c) one of the other two angles must measure 114 degrees
(d) one of the other two angles must measure 57 degrees
(e) one of the other two angles must measure either 66 degrees or 57 degrees

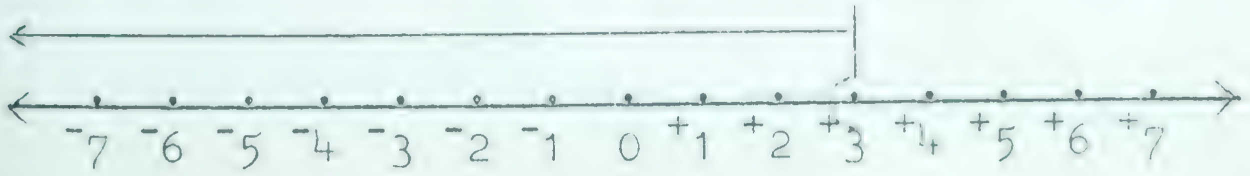
0. Which of the diagrams below best illustrates the set of real numbers that could be used as replacements for "x" to make a true statement of "2x + 1 is less than 3"?

(a)


(b)


(c)


(d)


(e)


21. Which of the following measurements is the most precise?

(a) 3.1 centimeters
(b) 310 millimeters
(c) 0.31 meters
(d) 3.10 centimeters
(e) 31 millimeters

2. If $^{+}2 + {}^{-}2 = 0$, then

(a) $^{+}6 + {}^{-}2 = {}^{-}8$

(b) $^{+}6 + {}^{-}2 = {}^{+}8$

(c) $^{+}6 + {}^{-}2 = {}^{+}4$

(d) $^{+}6 + {}^{-}2 = 0$

(e) $^{+}6 - {}^{-}2 = {}^{+}4$

3. Which of the following is not equivalent to 753?

(a) $7 \times 100 + 5 \times 10 + 3 \times 1$
(b) $6 \times 100 + 15 \times 10 + 3 \times 1$
(c) $7 \times 100 + 3 \times 10 + 23 \times 1$
(d) $6 \times 100 + 13 \times 10 + 23 \times 1$
(e) $7 \times 100 + 4 \times 10 + 23 \times 1$

4. Which of the following statements does not supply enough information for one to conclude that the geometric figures referred to are congruent.

(a) The figures could be placed together so that the points making up one figure would occupy exactly the same positions as the points making up the other figure.
(b) All corresponding parts of the figures have equal measures.
(c) All corresponding parts of the figures are congruent.
(d) The figures enclose exactly equal area measures.
(e) The figures have exactly the same size and shape.

. Suppose that a sculptor wants to make a scale model of a monument by reducing each measurement to 1/12 that of the original. The ratio of the volume of material that would be required for the scale model to that required for the monument would be

(a) 1/12
(b) 1/1728
(c) 1/36
(d) 1/144
(e) 12/1

26. The measure of the length of a rectangular solid is 7 feet, the measure of its width is 3 feet and the measure of its height is 4 feet. The area of its largest face would measure

(a) 21 square feet
(b) 28 square feet
(c) 49 square feet
(d) 84 square feet
(e) 12 square feet

7. Which of the following is not equivalent to the number one?

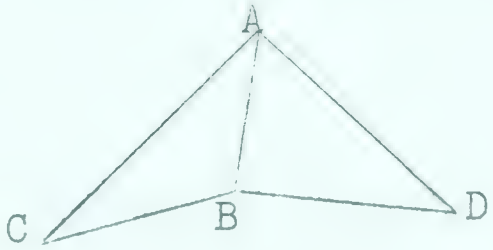
(a) $5 \div 5$
(b) $\frac{3}{4} \div \frac{3}{4}$
(c) $(4 + 1) \div (3 + 2)$
(d) $(3 - 1) \div (1 - 3)$
(e) $(2 \times 3) \div (5 + 1)$

8. Which of the following is correct?

(a) $3^4 = 3 \times 3 \times 3 \times 3$
(b) $4^3 = 3 \times 3 \times 3 \times 3$
(c) $2^3 = 2 \times 3$
(d) $5^4 = 5 + 5 + 5 + 5$
(e) $3^2 = 2 \times 2 \times 2$

. In the figure to the right below, line segment AB is made up of points

(a) that are on both line segment CB and line segment BD
(b) that are on both line segment AC and line segment AD
(c) that are on both triangle ABC and triangle ABD
(d) A and B
(e) that are on both line segment CA and line segment CB



. A rectangular playground measures 150 feet long by 50 feet wide. A fence is to be put around the two short sides and one long side of the playground. What would be the total cost of the fence at $3 per foot?

(a) $1200
(b) $1050
(c) $300
(d) $900
(e) $750

1. Twenty-seven cubes with one inch edges could be used to make

(a) a single cube 27 inches high, 27 inches long, 27 inches wide
(b) a single cube 9 inches high, 9 inches long, 9 inches wide
(c) a rectangular solid 1 inch high, 25 inches long, 1 inch wide
(d) a single cube 3 inches high, 3 inches long, 3 inches wide
(e) a rectangular solid 10 inches high, 10 inches long, 7 inches wide

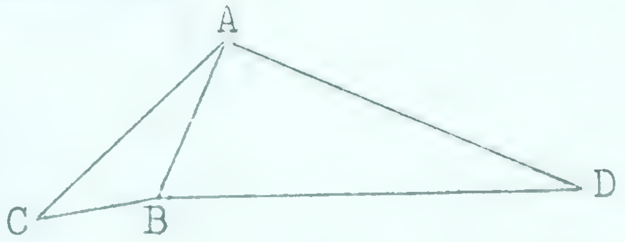
2. Which of the following means the same thing as $x = a \div b$, in which a, b and x represent numbers other than zero?

(a) $b \times x = a$
(b) $a \times x = b$
(c) $x \div b = a$
(d) $x = b \div a$
(e) $b \div x = a$

3. Which of the following represents the largest number?

(a) $3 \times 10^2 + 14 \times 10 + 5 \times 1$
(b) $4 \times 10^2 + 4 \times 10 + 5 \times 1$
(c) $3 \times 10^2 + 13 \times 10 + 25 \times 1$
(d) $4 \times 10^2 + 3 \times 10 + 15 \times 1$
(e) $2 \times 10^2 + 23 \times 10 + 5 \times 1$

. The best description of the set of points that lie both on line segment AB and on triangle ABD is



(a) point B
(b) point A
(c) line segment AD
(d) line segment BD
(e) line segment AB

. A merry-go-round has three rows of horses. The outer row is 6 feet farther out than the inner row. If you sit on an outside horse, how much farther do you ride in one turn of the merry-go-round than if you sit on an inside horse?

(a) 12π feet
(b) 6 feet
(c) 6π feet
(d) 12 feet
(e) Cannot be calculated from the information given.

36. Axel measured the capacity of a tank to be 100 cubic units. Bert measured the tank's capacity using a different cubic unit, and he found it to be 10 cubic units. Bert's cubic unit was ____________________ the size of the cubic unit Axel used.

(a) one tenth
(b) one hundred times
(c) one hundredth
(d) one thousand times
(e) ten times

37. If the replacement for "n" is always an odd number, then $n + 7$ is

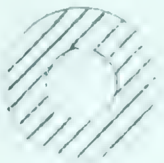
(a) always odd
(b) always divisible by 7
(c) any one of the natural numbers (counting numbers or zero)
(d) always even
(e) always less than 8

38. $10^4 \times 10^3 =$ ________________.

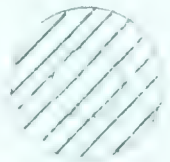
(a) $10 \times 10 \times 10 \times 10 \times 10$
(b) 10^7
(c) $10 \times 10 \times 10 \times 10^3$
(d) 10^{12}
(e) $10 \times 10 \times 10 \times 10 \times 10 \times 10$

39. In which of the following figures is the shaded region only that area that is in both the exterior of the smaller circle and the interior of the larger circle.

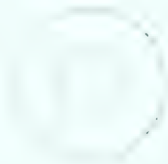
(a)



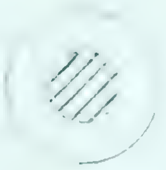
(b)



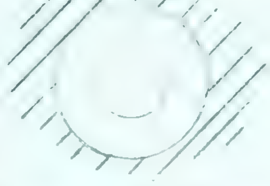
(c)



(d)



(e)



. I have "a" dollars. If I spend one-half of that amount, I will have less than seven dollars left. Which of the statements below describes this situation?

(a) 2a is greater than 7
(b) $\frac{1}{2}a = 7$
(c) $a - \frac{1}{2}a$ is greater than 7
(d) $a - \frac{1}{2}a$ is less than 7
(e) a is greater than 14

1. If both the length and width of a rectangular solid are doubled and the height is halved, the volume of the new solid is

(a) unchanged
(b) one-half that of the original solid
(c) twice that of the original solid
(d) four times that of the original solid
(e) eight times that of the original solid

2. If a positive integer is multiplied by a positive integer, the result is

(a) always even
(b) always odd
(c) always a positive integer
(d) never a positive integer
(e) sometimes a fraction

3. $10^5 \div 10^2 =$ ________________.

(a) $10^{2\frac{1}{2}}$

(b) $10 \times 10 \times 10 \times 10$

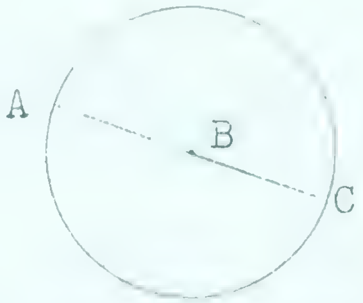
(c) 10^7

(d) 10×10

(e) 10^3

. In the figure to the right below, the best description of all the points that are both on line segment BC and on the circle

(a) is line segment BC
(b) is line segment AC
(c) is point B
(d) is point C
(e) are points A and C



. If a speed of 38 miles per hour is equivalent to a speed of 33 knots, which of the following statements would enable one to convert a speed in miles per hour to knots ("m" represents the number of miles per hour, and "k" represents the equivalent number of knots)?

(a) $38m = 33k$

(b) $\frac{k}{m} = \frac{38}{33}$

(c) $\frac{m}{k} = \frac{38}{33}$

(d) $\frac{38}{k} = \frac{33}{m}$

(e) $m - 5 = k$

6. A design is to be made by pasting coloured 2 inch by 2 inch squares side by side to cover an 8 inch by 12 inch rectangular area. How many 2 inch by 2 inch squares would be needed?

(a) 20
(b) 24
(c) 48
(d) 16
(e) 96

7. $^{+}7 + {}^{+}2({}^{-}3) =$ ______________.

(a) $^{+}1$

(b) $^{-}27$

(c) $^{+}13$

(d) $^{+}6$

(e) $^{+}2$

About how many tens are there in 5472?

(a) 5.5
(b) 5,472
(c) 54.7
(d) 54,720
(e) 547

If the radius of a circle is doubled, the area enclosed by the newly formed circle is

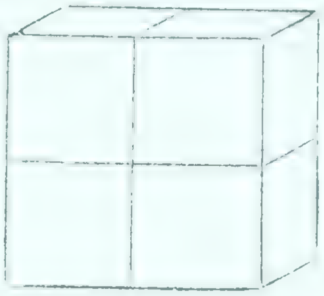
(a) unchanged
(b) one-half that enclosed by the original circle
(c) twice that enclosed by the original circle
(d) four times that enclosed by the original circle
(e) eight times that enclosed by the original circle

You are "x" years old and my age is twice yours. In three years the sum of our ages will be 27 years. Which mathematical sentence below represents this situation?

(a) $2x + x = 27$
(b) $2x + x = 24$
(c) $(2x - 3) + (x + 3) = 27$
(d) $(x - 3) + (2x - 3) = 27$
(e) $(x + 3) + (2x + 3) = 27$

1. The rectangular solid to the right below is made up of a number of cubes having equal volumes. If the volume of the rectangular solid is 7 cubic units, what is the volume of each of the cubes?

(a) 28 cubic units
(b) $\frac{4}{7}$ cubic units
(c) $\frac{8}{7}$ cubic units
(d) $\frac{7}{4}$ cubic units
(e) $\frac{7}{8}$ cubic units



2. If a, b and c represent numbers, which of the following statements is false?

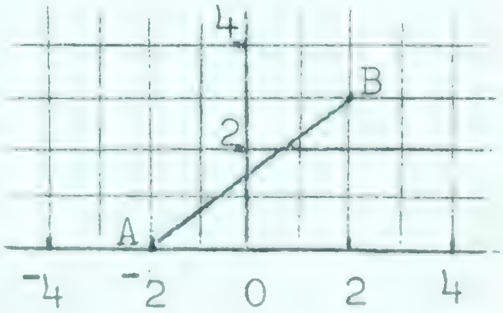
(a) For any two specific numbers, a and b, only one of the following can be true: a is less than b, a = b, a is greater than b.
(b) If a is less than b and b is less than c, then a must be less than c.
(c) If a is less than b and b is greater than c, then a must be less than c.
(d) If a is less than b, then a + c must be less than b + c.
(e) If a is less than b and c is greater than 0, then a X c must be less than b X c.

. Which of the following procedures would give you the correct product for 356 X 742?

(a) Add the products: 356 X 2; 356 X 4; 356 X 7
(b) Add the products: 356 X 7; 356 X 40; 356 X 200
(c) Add the products: 742 X 6; 742 X 5; 742 X 3
(d) Add the products: 742 X 6; 742 X 56; 742 X 300
(e) Add the products: 742 X 300; 742 X 50; 742 X 6

. The distance between points A and B in the diagram below

(a) is 4 units
(b) is 3 units
(c) is 25 units
(d) is 5 units
(e) cannot be determined without using a ruler, straight edge or pair of compasses.

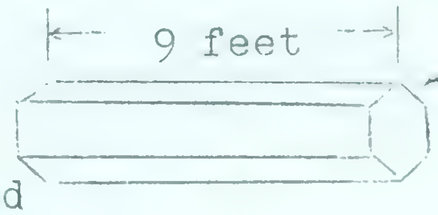


5. The current of a stream has a rate of "r" miles per hour. A motor boat goes downstream 10 miles per hour faster than the rate of the current. If the boat's speed downstream is 25 miles per hour, which mathematical statement below could be used to describe this situation?

(a) $r/10 = 2.5$
(b) $10/r = 25$
(c) $r + 10 = 25$
(d) $r - 10 = 25$
(e) $10r = 25$

. The volume of the figure to the right below

(a) is 3 cubic feet
(b) is 9 cubic feet
(c) is 81 cubic feet
(d) is 27 cubic feet
(e) cannot be calculated from the information given.



Uniform area of right cross-section: 3 square feet.

. Consider the following:

$$0.73 + 0.84 = (\frac{1}{100} \times 73) + (\frac{1}{100} \times 84)$$
$$= \frac{1}{100}(73 + 84)$$

Which of the following properties of a number system has been used in the steps shown above?

(a) $a + b = b + a$
(b) $ab = ba$
(c) $a + (b + c) = (a + b) + c$
(d) $a(bc) = (ab)c$
(e) $ab + ac = a(b + c)$

If 10^{-2} means $\frac{1}{10 \times 10}$ and 10^{-3} means $\frac{1}{10 \times 10 \times 10}$, then

$10^{-2} \times 10^{-3} =$ _______________.

(a) $\frac{1}{10 \times 5}$

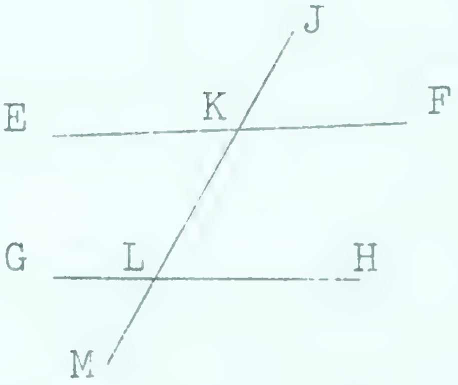
(b) 10^{5}

(c) $\frac{1}{10^{-5}}$

(d) 10^{-5}

(e) $\frac{1}{10}$

Which of the following statements about the figure below is false?



(a) Angle EKJ is adjacent to angle JKF.
(b) Angle KLH is adjacent to angle HLM.
(c) Angle FKL is adjacent to angle JKE.
(d) Angle GLK is adjacent to angle KLH.
(e) Angle EKJ is adjacent to angle EKL.

If "x" dollars is the cost per yard of a certain cloth, ten yards of cloth costs more than 12 dollars. Which statement below could be used to describe this situation?

(a) 10x is greater than 12
(b) x + 10 is greater than 12
(c) x is greater than 12
(d) 10/x is greater than 12
(e) x/10 is greater than 12

The measure of the width of a sand box is 3 feet and the measure of its length is 4 feet. How many cubic feet of sand would be needed to fill the box to a depth of 10 inches?

(a) 12
(b) 10
(c) 144
(d) 120
(e) 1440

Which of the following does not depend on the idea that $a + {}^{-}a = 0$?

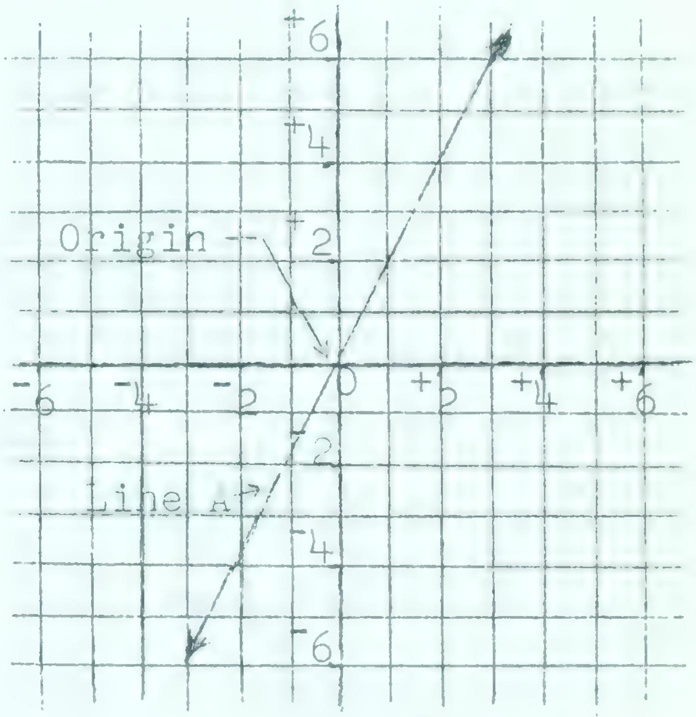
(a) $b + {}^{-}b = 0$
(b) ${}^{-}7 + {}^{+}7 = 0$
(c) ${}^{+}4 + {}^{+}3 + {}^{-}3 = {}^{+}4$
(d) $0 + {}^{-}4 = {}^{-}4$
(e) ${}^{+}36 + 6({}^{-}6) = 0$

63. Consider a numeration system in which only the following symbols are used: $ is 0, % is 1, ¢ is 2, & is 3. Which of the choices below would be the next three numerals in the sequence:

%, ¢, &, %$, %%, %¢, %&, ¢$, ...

(a) ¢¢, ¢&, &&
(b) ¢%, ¢¢, ¢&
(c) &$, &%, &¢
(d) ¢&, &$, &%
(e) &%, &¢, &&

64. Referring to the diagram to the right and counting "a" units horizontally from the origin and then "b" units vertically, for which of the following number replacements for "a" and "b" would you locate a point on line A?



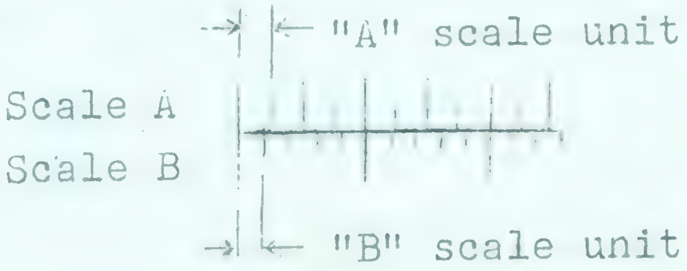
(a) $a = {}^{+}2,\ b = {}^{+}3$
(b) $a = {}^{-}2,\ b = {}^{+}4$
(c) $a = {}^{+}3,\ b = {}^{-}6$
(d) $a = {}^{+}1,\ b = {}^{+}2$
(e) $a = {}^{-}1,\ b = {}^{+}2$

5. In my pocket I have $3.90. If "n" represents the number of nickels I have and I have three times as many dimes as I have nickels and six more quarters than nickels, which of the following mathematical statements could be used to describe the situation?

(a) $n + 3n + n + 6 = 390$
(b) $5n + 6 = 390$
(c) $5n + 10(3n) + 25(n + 6) = 390$
(d) $5n + 10n + 25n + 6 = 390$
(e) $5n + 10(3n) + 25(6n) = 390$

6. Suppose that a line segment is measured with Scale A and then with Scale B. The ratio of the number of "A" scale units to the number of "B" scale units that would be used in making the measurements is

(a) 3/4
(b) 5/4
(c) 2/5
(d) 4/5
(e) 5/6



If $^{+}6 + {}^{-}6 = 0$ and $^{-}6 + \left[{}^{-}3({}^{-}2)\right] = 0$, then $^{-}3({}^{-}2)$ must be equivalent to

(a) $^{-}6$

(b) $^{+}6$

(c) $^{-}5$

(d) $^{+}5$

(e) 0

In the numeral 632, the 6 represents a value that is

(a) twice the value represented by the 3
(b) three times the value represented by the 2
(c) twenty times the value represented by the 3
(d) thirty times the value represented by the 2
(e) two hundred times the value represented by the 3

Which of the following statements is false?

(a) A circle with measurable radius can be drawn through any three points.
(b) A line segment can intersect a circle at one point, two points, or not at all.
(c) A line segment drawn through the center of a circle and having its end points on the circle is called a diameter of the circle.
(d) All of the points that make up a circle are at equal distances from the center of the circle.
(e) The interior of a circle does not include the circle.

Given the statement: $3a = 4b$, consider which of the following situations this statement could represent.

(1) My salary quadrupled is matched by Jack's salary tripled.
(2) There are four more bananas when the number of apples is increased by three.
(3) The perimeters of an equilateral triangle and a square have equal measures.
(4) Three times a certain number is equal to four times another number.

$3a = 4b$ could represent

(a) All four situations described above.
(b) Only (1), (3) and (4)
(c) Only (2), (3) and (4)
(d) Only (2)
(e) Only (4)

71. A square, a rectangle and an eqilateral triangle have equal perimeter measures. Which of the following statements about them is true?

    (a) The square and the rectangle enclose equal areas.
    (b) The square and the triangle enclose equal areas.
    (c) The area enclosed by the triangle is one-half that enclosed by the rectangle.
    (d) The area enclosed by the square is greater than that enclosed by either of the other two figures.
    (e) The area enclosed by the rectangle is greater than that enclosed by either of the other two figures.

72. If the symbols @, #, $, % represent numbers arranged in order from smallest to largest, which of the following statements is true?

    (a) $ represents a number larger than @ does but smaller than # does.
    (b) # represents a number smaller than % does but larger than @ does.
    (c) @ represents a number smaller than % does but larger than # does.
    (d) % represents a number larger than $ does but smaller than # does.
    (e) @ represents a number smaller than # does but larger than $ does.

73. Imagine a place where the inhabitants have only 6 fingers altogether so that they use six in their system of numeration just as we use 10 in ours. For instance, to them "23" means 2 groups of 6 and 3 units. What numeral would be used in such a system to represent the number of marks in the set to the right below?

    (a) 23
    (b) 21
    (c) 33
    (d) 43
    (e) 29

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* * * * * *
* * * * * *
* * * * * *
* * *
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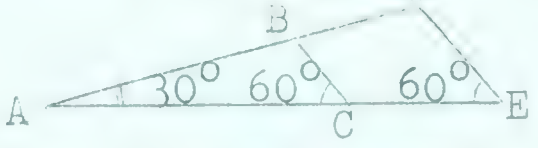
4. In the figure below, the measure of line segment BC

    (a) is 4 inches
    (b) is 3 inches
    (c) is 5 inches
    (d) is 4½ inches
    (e) cannot be determined from the information given.



Line Segment	Measure
AB	6 inches
DE	6 inches
AD	9 inches

75. An increase of 20% in Mr. Jones' present income ("m" dollars per month) would give him an income of $300.00 per month. Which of the following statements would enable you to find his correct present monthly income?

(a) $\frac{m}{300} = \frac{20}{100}$

(b) $\frac{300}{m} = \frac{120}{100}$

(c) $\frac{300 - m}{300} = \frac{20}{100}$

(d) $\frac{m}{300} = \frac{120}{100}$

(e) $m + 20 = 300$

ITEM DISCRIMINATION AND DIFFICULTY INDICES FOR PILOT STUDY VERSION OF SMU TEST

	MEASUREMENT				NUMBER SYSTEMS				NUMERATION				GEOMETRY				PROBLEM SOLVING			
	n_p	r	p	n_f	n_p	r	p	n_f	n_p	r	p	n_f	n_p	r	p	n_f	n_p	r	p	n_f
ITEMS CHOSEN FOR FINAL FORM	26	.62	.54	16	37	.73	.78	17	68	.73	.46	43	39	.73	.60	14	25	1.00	.11	50
	41	.61	.67	11	52	.64	.62	32	43	.66	.76	18	4	.64	.65	4	55	.68	.67	15
	61	.58	.30	36	32	.63	.49	42	53	.44	.54	28	29	.61	.61	9	60	.65	.81	5
	31	.58	.49	26	42	.60	.85	2	33	.44	.78	13	44	.60	.30	44	10	.61	.75	10
	6	.50	.75	1	17	.53	.78	12	63	.43	.47	38	49	.59	.44	29	50	.55	.62	20
	51	.47	.54	21	47	.49	.78	22	73	.40	.54	33	59	.50	.49	24	70	.55	.49	25
	46	.42	.73	6	72	.49	.78	27	58	.34	.26	48	69	.48	.33	39	20	.43	.43	30
	71	.40	.12	46	7	.42	.83	7	38	.33	.83	3	9	.44	.40	34	65	.37	.39	40
	66	.33	.35	31	62	.40	.45	47	48	.32	.81	8	24	.41	.51	19	45	.32	.42	35
	11	.20	.28	41	57	.32	.52	37	13	.30	.59	23	64	.30	.28	49	75	.30	.25	45
DISCARDED ITEMS	1	.20	.84		2	.40	.79		23	N	.88		54	.30	.14		40	.42	.83	
	36	.13	.31		67	.37	.84		8	N	.91		34	.30	.77		5	.37	.59	
	56	.13	.33		12	.15	.20		18	N	.91		14	.04	.78		30	N	.09	
	21	.07	.22		27	.07	.87		3	N	.91		74	N	.08		35	.30	.77	
	16	N	.12		22	N	.86		28	N	.93		19	N	.20		15	N	.96	

n_p = item number on pilot study version of the test

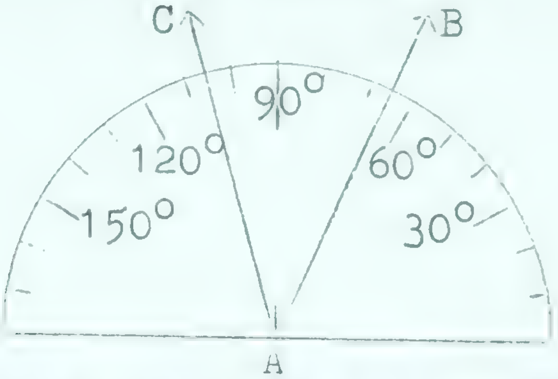
r = tetrachoric r (index of discrimination)

p = proportion of students in pilot study who passed item (index of difficulty)

n_f = item number on final version of test

FINAL VERSION OF SMU TEST

1. The measure of angle BAC is approximately



(a) 65°
(b) 105°
(c) 40°
(d) 50°
(e) 30°

2. If a positive integer is multiplied by a positive integer, the result is

(a) always even
(b) always odd
(c) always a positive integer
(d) never a positive integer
(e) sometimes a fraction

3. $10^4 \times 10^3 =$ ________________.

(a) $10 \times 10 \times 10 \times 10 \times 10 \times 10$
(b) 10^7
(c) $10 \times 10 \times 10 \times 10^3$
(d) 10^{12}
(e) $10 \times 10 \times 10 \times 10 \times 10$

4. If the measure of one angle of a scalene triangle is 40 degrees, which of the following statements is always true?

(a) The measure of one of the other angles is 90 degrees.
(b) The measure of one of the other angles is 40 degrees.
(c) The sum of the measures of the other two angles is 140 degrees.
(d) Two of the sides have equal measures.
(e) The measure of one of the other angles is 140 degrees.

5. If "x" dollars is the cost per yard of a certain cloth, ten yards of cloth costs more than 12 dollars. Which statement below could be used to describe this situation?

(a) 10x is greater than 12
(b) x + 10 is greater than 12
(c) x is greater than 12
(d) 10/x is greater than 12
(e) x/10 is greater than 12

6. A design is to be made by pasting coloured 2 inch by 2 inch squares side by side to cover an 8 inch by 12 inch rectangular area. How many 2 inch by 2 inch squares would be needed?

(a) 20
(b) 24
(c) 48
(d) 16
(e) 96

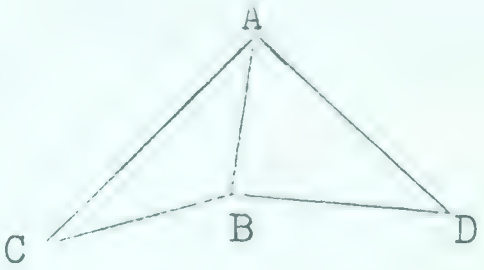
7. In which of the following is 36 completely factored (expressed as the product of primes)?

(a) 4 X 9
(b) 2 X 3 X 6
(c) 3 X 12
(d) 2 X (9 + 9)
(e) 2 X 2 X 3 X 3

8. About how many tens are there in 5472?

(a) 5.5
(b) 5,472
(c) 54.7
(d) 54,720
(e) 547

9. In the figure to the right below, line segment AB is made up of points



(a) that are on both line segment CB and line segment BD
(b) that are on both line segment AC and line segment AD
(c) that are on both triangle ABC and triangle ABD
(d) A and B
(e) that are on both line segment CA and line segment CB

10. A father is nine times as old as his son. If the son is now "n" years old, how old will the father be in 8 years?

(a) 9n + 8 years
(b) 9(n + 8) years
(c) n + 8 years
(d) n + 9(8) years
(e) 9n years

11. If both the length and the width of a rectangular solid are doubled and the height is halved, the volume of the new solid is

(a) that of the original solid
(b) one-half that of the original solid
(c) twice that of the original solid
(d) four times that of the original solid
(e) eight times that of the original solid

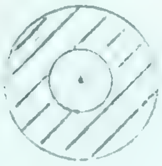
12. If a, b and c represent numbers, which of the following is not correct?

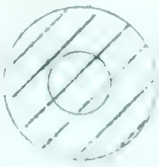
(a) $ab = ba$
(b) $a(b + c) = ab + ac$
(c) $a + (b + c) = (a + b) + c$
(d) $a + (b \times c) = (a + b) \times (a + c)$
(e) $a + b = b + a$

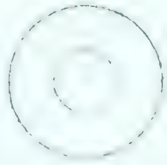
13. Which of the following represents the largest number?

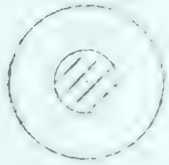
(a) $3 \times 10^2 + 14 \times 10 + 5 \times 1$
(b) $4 \times 10^2 + 4 \times 10 + 5 \times 1$
(c) $4 \times 10^2 + 3 \times 10 + 15 \times 1$
(d) $2 \times 10^2 + 23 \times 10 + 5 \times 1$
(e) $3 \times 10^2 + 13 \times 10 + 25 \times 1$

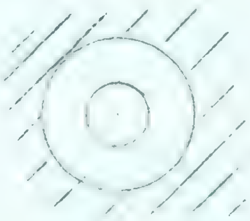
14. In which of the following figures is the shaded region only that area that is in both the exterior of the smaller circle and the interior of the larger circle?

(a)


(b)


(c)


(d)


(e)


15. The current of a stream has a rate of "r" miles per hour. A motor boat goes downstream 10 miles per hour faster than the rate of the current. If the boat's speed downstream is 25 miles per hour, which mathematical statement below could be used to describe this situation?

(a) $r/10 = 2.5$
(b) $10/r = 25$
(c) $r + 10 = 25$
(d) $r - 10 = 25$
(e) $10r = 25$

16. The measure of the length of a rectangular solid is 7 feet, the measure of its width is 3 feet and the measure of its height is 4 feet. The area of its largest face would measure

(a) 21 square feet
(b) 28 square feet
(c) 49 square feet
(d) 84 square feet
(e) 12 square feet

17. If the replacement for "n" is always an odd number, then n + 7 is

(a) always odd
(b) always divisible by 7
(c) any one of the natural numbers (counting numbers or zero)
(d) always even
(e) always less than 8

18. $10^5 \div 10^2 =$ ________________.

(a) $10^{2\frac{1}{2}}$

(b) $10 \times 10 \times 10 \times 10$

(c) 10^7

(d) 10×10

(e) 10^3

19. Which of the following statements does not supply enough information for one to conclude that the geometric figures referred to are congruent.

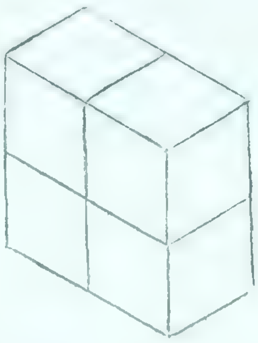
(a) The figures could be put together so that the points making up one figure would occupy exactly the same positions as the points making up the other figure.
(b) All corresponding parts of the figures have equal measures.
(c) All corresponding parts of the figures are congruent.
(d) The figures enclose exactly equal area measures.
(e) The figures have exactly the same size and shape.

20. You are "x" years old and my age is twice yours. In three years the sum of our ages will be 27 years. Which mathematical sentence below represents this situation?

(a) $2x + x = 27$
(b) $2x + x = 24$
(c) $(2x - 3) + (x + 3) = 27$
(d) $(x - 3) + (2x - 3) = 27$
(e) $(x + 3) + (2x + 3) = 27$

21. The rectangular solid to the right below is made up of a number of cubes having equal volumes. If the volume of the rectangular solid is 7 cubic units, what is the volume of each of the cubes?

(a) 28 cubic units
(b) $\frac{4}{7}$ cubic units
(c) $\frac{8}{7}$ cubic units
(d) $\frac{7}{4}$ cubic units
(e) $\frac{7}{8}$ cubic units



22. $^{+}7 + {}^{+}2(^{-}3) =$ _______________.

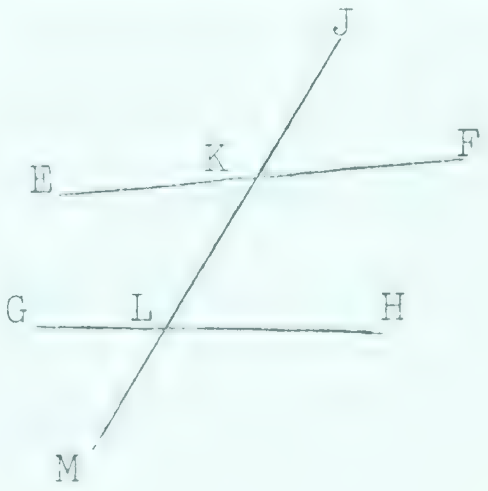
(a) $^{+}1$
(b) $^{-}27$
(c) $^{+}13$
(d) $^{+}6$
(e) $^{+}2$

23. If N represents any natural number (counting number or zero), which of the following will always represent an odd number?

(a) N
(b) N + 1
(c) N + 2
(d) 2N + 2
(e) 2N + 1

24. Which of the following statements about the figure to the right below is false?

(a) Angle EKJ is adjacent to angle JKF.
(b) Angle KLH is adjacent to angle HLM.
(c) Angle FKL is adjacent to angle JKE.
(d) Angle GLK is adjacent to angle KLH.
(e) Angle EKJ is adjacent to angle EKL.



25. Given the statement: $3a = 4b$, consider which of the following situations this statement could represent.

   (1) My salary quadrupled is matched by Jack's salary tripled.
   (2) There are four more bananas when the number of apples is increased by three.
   (3) The measures of the perimeters of an equilateral triangle and a square are equal.
   (4) Three times a certain number is equal to four times another number.

   $3a = 4b$ could represent

   (a) all four situations described above.
   (b) only (1), (3) and (4)
   (c) only (2), (3) and (4)
   (d) only (2)
   (e) only (4)

26. Twenty-seven cubes with one inch edges could be used to make

   (a) a single cube 27 inches high, 27 inches long, 27 inches wide
   (b) a single cube 9 inches high, 9 inches long, 9 inches wide
   (c) a rectangular solid 1 inch high, 25 inches long, 1 inch wide
   (d) a single cube 3 inches high, 3 inches long, 3 inches wide
   (e) a rectangular solid 10 inches high, 10 inches long, 7 inches wide

27. If the symbols @, #, $, % represent numbers arranged in order from smallest to largest, which of the following statements is true?

   (a) $ represents a number larger than @ does but smaller than # does.
   (b) # represents a number smaller than % does but larger than @ does.
   (c) @ represents a number smaller than % does but larger than # does.
   (d) % represents a number larger than $ does but smaller than # does.
   (e) @ represents a number smaller than # does but larger than $ does.

28. Which of the following procedures would give you the correct product for 356 X 742?

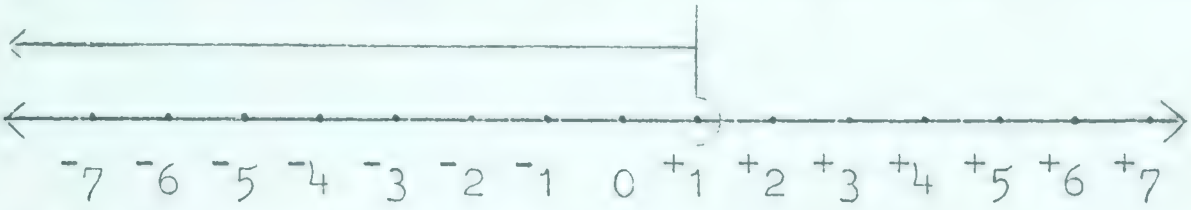
   (a) Add the products: 356 X 2; 356 X 4; 356 X 7
   (b) Add the products: 356 X 7; 356 X 40; 356 X 200
   (c) Add the products: 742 X 6; 742 X 5; 742 X 3
   (d) Add the products: 742 X 6; 742 X 56; 742 X 300
   (e) Add the products: 742 X 300; 742 X 50; 742 X 6

29. If the radius of a circle is doubled, the area enclosed by the newly formed circle is

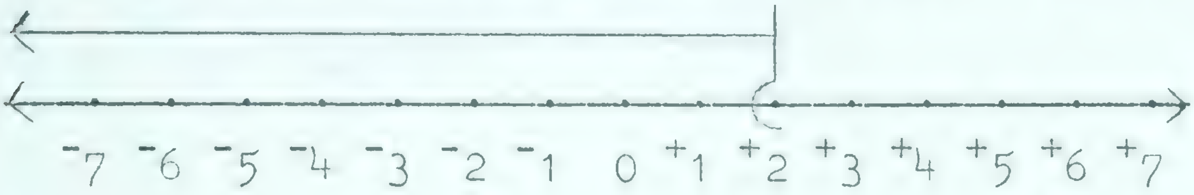
(a) equal to that enclosed by the original circle
(b) one-half that enclosed by the original circle
(c) twice that enclosed by the original circle
(d) four times that enclosed by the original circle
(e) eight times that enclosed by the original circle

30. Which of the diagrams below best illustrates the set of real numbers that could be used as replacements for "x" to make a true statement of "2x + 1 is less than 3"?

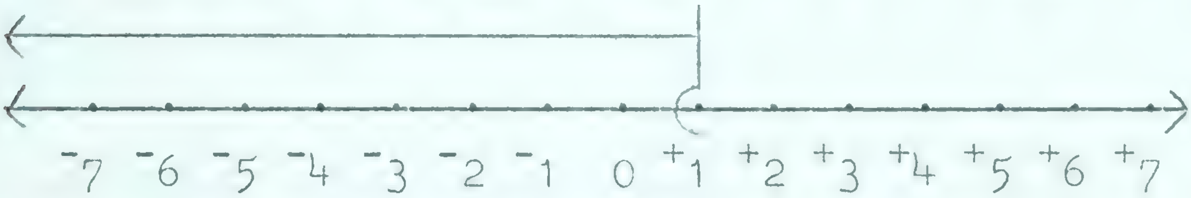
(a)



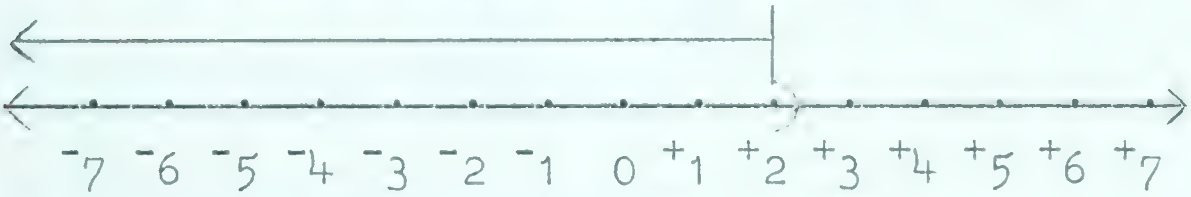
(b)



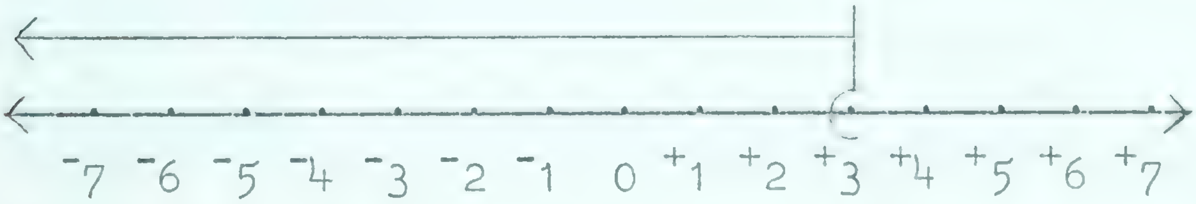
(c)



(d)

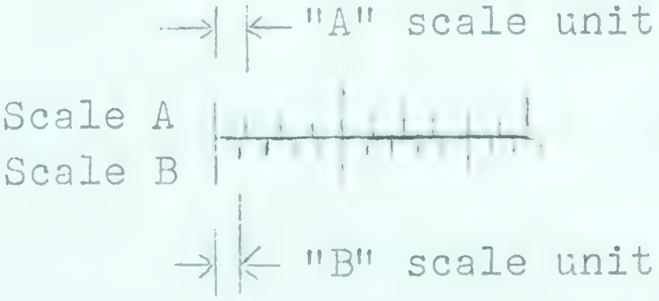


(e)



31. Suppose that a line segment is measured with Scale A and then with Scale B. The ratio of the number of "A" scale units to the number of "B" scale units that would be used in making the measurements is

(a) 3/4
(b) 5/4
(c) 2/5
(d) 4/5
(e) 5/6



32. If a, b and c represent numbers, which of the following statements is false?

(a) For any two specific numbers, a and b, only one of the following can be true: a is less than b, $a = b$, a is greater than b.
(b) If a is less than b and b is less than c, then a must be less than c.
(c) If a is less than b and b is greater than c, then a must be less than c.
(d) If a is less than b, then $a + c$ must be less than $b + c$.
(e) If a is less than b and c is greater than 0, then $a \times c$ must be less than $b \times c$.

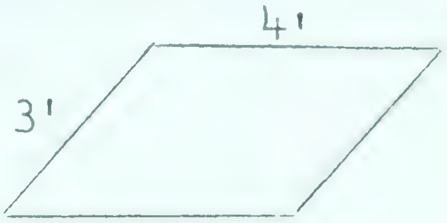
33. Imagine a place where the inhabitants have only 6 fingers altogether so that they use six in their system of numeration just as we use ten in ours. For instance, to them "23" means 2 groups of 6 and 3 units. What numeral would be used in such a system to represent the number of marks in the set to the right below?

(a) 33
(b) 23
(c) 21
(d) 43
(e) 29

* * * * * *
* * * * * *
* * * * * *
* * *

34. The area of the parallelogram shown at the right below

(a) can be found by adding 3 and 4
(b) can be found by multiplying 3 by 4
(c) can be found by multiplying 3 by 4 and dividing the product by 2
(d) can be found by multiplying $(3 + 4)$ by $\frac{1}{2}$
(e) cannot be found from the information given



35. If a speed of 38 miles per hour is equivalent to a speed of 33 knots, which of the following statements would enable one to convert a speed in miles per hour to knots ("m" represents the number of miles per hour, and "k" represents the equivalent number of knots)?

(a) $38m = 33k$
(b) $\frac{k}{m} = \frac{38}{33}$
(c) $\frac{m}{k} = \frac{38}{33}$
(d) $\frac{38}{k} = \frac{33}{m}$
(e) $m - 5 = k$

36. The measure of the width of a sand box is 3 feet and the measure of its length is 4 feet. How many cubic feet of sand would be needed to fill the box to a depth of 10 inches?

(a) 12
(b) 10
(c) 144
(d) 120
(e) 1440

37. Consider the following:

$$0.73 + 0.84 = (\frac{1}{100} \times 73) + (\frac{1}{100} \times 84)$$
$$= \frac{1}{100}(73 + 84)$$

Which of the following properties of a number system has been used in the steps shown above?

(a) $a + b = b + a$
(b) $ab = ba$
(c) $a + (b + c) = (a + b) + c$
(d) $a(bc) = (ab)c$
(e) $ab + ac = a(b + c)$

38. Consider a numeration system in which only the following symbols are used: $ is 0, % is 1, ¢ is 2, & is 3. Which of the choices below would be the next three numerals in the sequence:

%, ¢, &, %$, %%, %¢, %&, ¢$, . . .

(a) ¢¢, ¢&, &$
(b) ¢%, ¢¢, ¢&
(c) &$, &%, &¢
(d) ¢&, &$, &%
(e) &%, &¢, &&

39. Which of the following statements is false?

(a) A circle with measurable radius can be drawn through any three points.
(b) A line segment can intersect a circle at one point, two points, or not at all.
(c) A line segment drawn through the center of a circle and having its end points on the circle is called a diameter of the circle.
(d) All of the points that make up a circle are at equal distances from the center of the circle.
(e) The interior of a circle does not include the circle.

40. In my pocket I have \$3.90. If "n" represents the number of nickels I have and I have three times as many dimes as I have nickels, and six more quarters than nickels, which of the following mathematical statements could be used to describe the situation?

(a) $n + 3n + n + 6 = 390$
(b) $5n + 6 = 390$
(c) $5n + 10(3n) + 25(n + 6) = 390$
(d) $5n + 10n + 25n + 6 = 390$
(e) $5n + 10(3n) + 25(6n) = 390$

41. If the measurement of the length of a line segment is recorded as $2\frac{3}{8}$ inches, it is assumed that the true measure of the length of the line segment

(a) is $2\frac{3}{8}$ inches
(b) could be as small as $2\frac{2}{8}$ inches or as large as $2\frac{4}{8}$ inches
(c) could be as small as $2\frac{1}{8}$ inches or as large as $2\frac{5}{8}$ inches
(d) could be as small as $2\frac{5}{16}$ inches or as large as $2\frac{7}{16}$ inches
(e) could be as small as $2\frac{3}{16}$ inches or as large as $2\frac{9}{16}$ inches

42. Which of the following means the same thing as $x = a \div b$, in which a, b and x represent numbers other than zero?

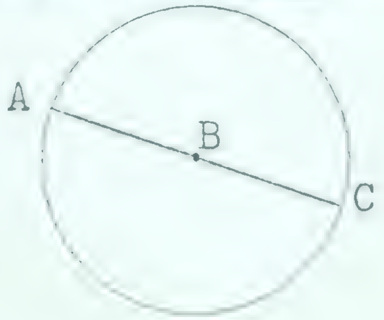
(a) $b \times x = a$
(b) $a \times x = b$
(c) $x \div b = a$
(d) $x = b \div a$
(e) $b \div x = a$

43. In the numeral 632, the 6 represents a value that is

(a) twice the value represented by the 3
(b) three times the value represented by the 2
(c) twenty times the value represented by the 3
(d) thirty times the value represented by the 2
(e) two hundred times the value represented by the 3

44. In the figure to the right below, the best description of all the points that are both on line segment BC and on the circle

(a) is line segment BC
(b) is line segment AC
(c) is point B
(d) is point C
(e) are points A and C



45. An increase of 20% in Mr. Jones' present income ("m" dollars per month) would give him an income of $300.00 per month. Which of the following statements would enable you to find his correct present monthly income?

(a) $\frac{m}{300} = \frac{20}{100}$

(b) $\frac{300}{m} = \frac{120}{100}$

(c) $\frac{300 - m}{300} = \frac{20}{100}$

(d) $\frac{m}{300} = \frac{120}{100}$

(e) $m + 20 = 300$

46. The measures of the perimeters of a square, a rectangle and an equilateral triangle are equal. Which of the following statements about them is true?

(a) The area enclosed by the square is greater than that enclosed by either of the other two figures.
(b) The square and the rectangle enclose equal areas.
(c) The square and the triangle enclose equal areas.
(d) The area enclosed by the triangle is one-half that enclosed by the rectangle.
(e) The area enclosed by the rectangle is greater than that enclosed by either of the other two figures.

47. Which of the following does not depend on the idea that $a + {}^-a = 0$?

(a) $b + {}^-b = 0$

(b) ${}^-7 + {}^+7 = 0$

(c) ${}^+4 + {}^+3 + {}^-3 = {}^+4$

(d) $0 + {}^-4 = {}^-4$

(e) ${}^+36 + 6({}^-6) = 0$

48. If 10^{-2} means $\frac{1}{10 \times 10}$ and 10^{-3} means $\frac{1}{10 \times 10 \times 10}$, then $10^{-2} \times 10^{-3} =$ ________________.

(a) $\frac{1}{10 \times 5}$

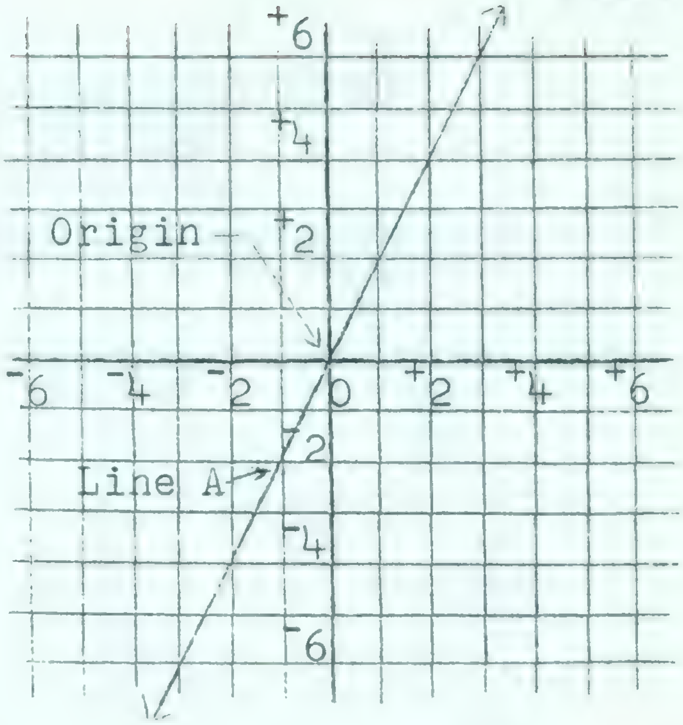
(b) 10^5

(c) $\frac{1}{10^{-5}}$

(d) $\frac{1}{10}$

(e) 10^{-5}

49. Referring to the diagram to the right and counting "a" units horizontally from the origin and then "b" units vertically, for which of the following number replacements for "a" and "b" would you locate a point on line A?



(a) $a = {}^{+}2,\ b = {}^{+}3$

(b) $a = {}^{-}2,\ b = {}^{+}4$

(c) $a = {}^{+}3,\ b = {}^{-}6$

(d) $a = {}^{+}1,\ b = {}^{+}2$

(e) $a = {}^{-}1,\ b = {}^{+}2$

50. Suppose that a sculptor wants to make a scale model of a monument by reducing each measurement to 1/12 that of the original. The ratio of the volume of material that would be required for the scale model to that required for the monument would be

(a) 1/12
(b) 1/1728
(c) 1/36
(d) 1/144
(e) 12/1

DATA FOR CALCULATION OF RELIABILITY INDEX FOR FINAL FORM OF SMU TEST FROM MARCH, 1964 TESTING

Item Number g	Proportion of Sample that Answered Item Correctly p_g	Item Number g	Proportion of Sample that Answered Item Correctly p_g
1	.4768	26	.2003
2	.6970	27	.6837
3	.6920	28	.4304
4	.4519	29	.1109
5	.5447	30	.3807
6	.5165	31	.2814
7	.7086	32	.3990
8	.6771	33	.4205
9	.5231	34	.3228
10	.6374	35	.4470
11	.3460	36	.1440
12	.5132	37	.3725
13	.4569	38	.3824
14	.5281	39	.3559
15	.4900	40	.1771
16	.3592	41	.0894
17	.6672	42	.3112
18	.5678	43	.2847
19	.3145	44	.0778
20	.3675	45	.1672
21	.3327	46	.1258
22	.2483	47	.2069
23	.5811	48	.1639
24	.4354	49	.1771
25	.2913	50	.5960

Number of Items, $K = 50$

Test Score Variance, $S_x^2 = 55.1183$

$$\text{Reliability Index} = \left(\frac{K}{K-1}\right)\left[1 - \frac{\sum_{g=1}^{K} p_g(1-p_g)}{S_x^2}\right]$$

RAW SCORES FOR ALL STUDENTS

Beginning on the following page, the scores obtained by the *Seeing Through Mathematics* students are listed.

The symbols heading the columns are to be interpreted as follows:

AC (Column 1): September Iowa Arithmetic Concepts Score

PS (Column 2): September Iowa Problem Solving Score

V (Column 3): September SCAT Verbal Ability Score

Q (Column 4): September SCAT Quantitative Ability Score

SMU (Column 5): March SMU Test Score

AC (Column 6): March Iowa Arithmetic Concepts Score

PS (Column 7): March Iowa Problem Solving Score

TOT (Column 8): Total SCAT Score (V + Q)

ID (Column 9): Identification Number

AC	PS	V	Q	SMU	AC	PS	TOT	ID
26.	20.	51.	39.	18.	37.	23.	90	1001
21.	15.	44.	38.	15.	28.	10.	82	1002
23.	19.	45.	40.	11.	28.	13.	85	1003
17.	7.	41.	40.	15.	33.	14.	81	1004
13.	4.	29.	31.	22.	15.	8.	60	1005
18.	12.	50.	35.	18.	27.	11.	85	1006
13.	14.	37.	37.	10.	17.	10.	74	1007
16.	12.	41.	38.	11.	26.	13.	79	1008
14.	6.	50.	28.	14.	22.	9.	78	1009
16.	9.	50.	27.	16.	20.	11.	77	1010
23.	11.	46.	37.	14.	25.	14.	83	1011
29.	10.	51.	43.	21.	36.	19.	94	1012
17.	12.	41.	36.	9.	17.	14.	77	1013
18.	14.	47.	39.	13.	23.	21.	86	1014
23.	11.	49.	41.	23.	36.	16.	90	1015
21.	10.	50.	44.	14.	31.	10.	94	1016
27.	14.	49.	42.	13.	29.	18.	91	1017
23.	13.	43.	30.	14.	30.	17.	73	1018
40.	24.	57.	50.	32.	42.	21.	107	1019
19.	12.	48.	30.	9.	22.	7.	78	1020
31.	18.	55.	40.	29.	34.	18.	95	1021
41.	24.	58.	48.	43.	46.	26.	106	1022
21.	6.	49.	28.	11.	22.	12.	77	1023
39.	22.	57.	47.	31.	43.	25.	104	1024
28.	16.	50.	44.	20.	27.	19.	94	1025
19.	15.	53.	42.	16.	35.	22.	95	1026
32.	20.	56.	41.	37.	39.	20.	97	1027
22.	9.	59.	31.	22.	25.	12.	90	1028
10.	12.	30.	25.	9.	12.	10.	55	1029
27.	14.	48.	43.	23.	38.	15.	91	1030
27.	16.	38.	40.	18.	31.	17.	78	1031
20.	9.	54.	28.	15.	18.	16.	82	1032
31.	17.	53.	46.	25.	42.	21.	99	1033
25.	14.	53.	40.	25.	33.	12.	93	1034

AC	PS	V	Q	SMU	AC	PS	TOT	ID
29.	19.	53.	48.	26.	39.	24.	101	1035
24.	5.	52.	36.	16.	30.	12.	88	1036
17.	9.	39.	38.	16.	26.	8.	77	1037
27.	17.	49.	45.	21.	37.	16.	94	1038
15.	10.	41.	34.	20.	28.	17.	75	1039
14.	12.	37.	33.	10.	16.	17.	70	1040
18.	14.	41.	39.	16.	27.	17.	80	1041
19.	18.	49.	36.	16.	17.	12.	85	1042
40.	24.	46.	47.	36.	41.	26.	93	1043
32.	15.	49.	43.	17.	31.	15.	92	1044
12.	15.	43.	30.	14.	17.	13.	73	1045
12.	12.	42.	23.	9.	17.	10.	65	1046
23.	13.	44.	36.	21.	32.	19.	80	1047
20.	8.	45.	36.	12.	30.	15.	81	1048
17.	12.	51.	44.	18.	26.	17.	95	1049
31.	16.	51.	44.	24.	37.	22.	95	1050
39.	11.	58.	46.	23.	36.	19.	104	1051
21.	18.	41.	34.	18.	21.	14.	75	1052
32.	18.	49.	46.	22.	38.	26.	95	1053
32.	10.	53.	43.	25.	37.	21.	96	1054
39.	21.	56.	50.	40.	41.	31.	106	1055
30.	22.	49.	49.	31.	37.	28.	98	1056
33.	14.	50.	45.	19.	34.	13.	95	1057
38.	23.	52.	48.	30.	35.	22.	100	1058
36.	22.	52.	47.	31.	46.	28.	99	1059
30.	18.	54.	50.	24.	33.	28.	104	1060
32.	21.	49.	44.	14.	25.	23.	93	1061
26.	11.	45.	46.	23.	34.	22.	91	1062
25.	11.	50.	34.	16.	27.	20.	84	1063
13.	11.	53.	38.	17.	24.	15.	91	1064
29.	13.	53.	35.	19.	35.	19.	88	1065
39.	12.	35.	46.	24.	36.	18.	81	1066
28.	15.	58.	45.	21.	33.	22.	98	1067
[illegible]	8.	55.	42.	16.	30.	17.	97	1068

AC	PS	V	Q	SMU	AC	PS	TOT	ID
38.	15.	59.	47.	34.	43.	27.	106	1069
18.	12.	41.	40.	25.	29.	22.	81	1070
34.	16.	34.	45.	22.	33.	14.	79	1071
24.	13.	49.	37.	19.	28.	17.	86	1072
30.	21.	44.	38.	25.	40.	20.	82	1073
36.	17.	57.	43.	27.	42.	21.	100	1074
24.	13.	47.	41.	14.	23.	22.	88	1075
16.	16.	41.	40.	16.	26.	20.	81	1076
32.	17.	53.	46.	23.	35.	17.	99	1077
39.	28.	58.	50.	44.	46.	26.	108	1078
37.	9.	52.	44.	23.	40.	19.	96	1079
29.	22.	52.	44.	27.	38.	25.	96	1080
21.	19.	52.	41.	13.	25.	16.	93	1081
37.	16.	54.	49.	33.	37.	22.	103	1082
25.	15.	49.	43.	30.	34.	20.	92	1083
38.	21.	55.	45.	31.	41.	27.	100	1084
30.	16.	57.	42.	19.	34.	14.	99	1085
33.	18.	57.	46.	41.	39.	20.	103	1086
36.	19.	56.	47.	31.	42.	19.	103	1087
24.	12.	55.	42.	25.	37.	13.	97	1088
26.	18.	57.	46.	32.	30.	23.	103	1089
41.	22.	56.	50.	38.	46.	21.	106	1090
27.	23.	48.	48.	28.	38.	21.	96	1091
28.	17.	61.	46.	21.	38.	20.	107	1092
32.	17.	52.	42.	24.	37.	19.	94	1093
35.	18.	51.	50.	31.	42.	20.	101	1094
32.	17.	56.	46.	19.	36.	23.	102	1095
29.	17.	55.	41.	23.	33.	17.	96	1096
35.	20.	57.	48.	30.	40.	22.	105	1097
35.	22.	54.	46.	30.	41.	24.	100	1098
30.	16.	50.	43.	27.	32.	16.	93	1099
30.	14.	55.	44.	20.	40.	16.	99	1100
31.	20.	50.	41.	25.	32.	13.	91	1101
33.	24.	44.	46.	17.	41.	19.	90	1102

AC	PS	V	Q	SMU	AC	PS	TOT	ID
31.	18.	55.	49.	30.	38.	12.	104	1103
28.	6.	43.	41.	15.	28.	9.	84	1104
36.	17.	54.	47.	25.	42.	21.	101	1105
31.	13.	51.	41.	18.	26.	22.	92	1106
38.	23.	55.	43.	29.	41.	25.	98	1107
25.	15.	55.	41.	26.	33.	18.	96	1108
34.	19.	49.	43.	23.	36.	18.	92	1109
31.	18.	40.	31.	15.	33.	25.	71	1110
31.	17.	51.	46.	28.	37.	22.	97	1111
32.	17.	55.	44.	26.	41.	19.	99	1112
37.	24.	53.	50.	32.	37.	27.	103	1113
35.	22.	50.	47.	26.	39.	26.	97	1114
23.	16.	46.	42.	25.	32.	26.	88	1115
21.	20.	51.	46.	23.	33.	24.	97	1116
39.	24.	58.	49.	39.	43.	30.	107	1117
35.	20.	56.	46.	25.	37.	21.	102	1118
39.	19.	57.	48.	28.	45.	25.	105	1119
36.	19.	56.	46.	31.	38.	24.	102	1120
43.	25.	57.	50.	38.	45.	28.	107	1121
38.	14.	56.	44.	21.	39.	22.	100	1122
15.	12.	47.	38.	23.	27.	23.	85	1123
29.	15.	50.	46.	22.	34.	23.	96	1124
38.	26.	52.	46.	23.	35.	28.	98	1125
23.	22.	53.	48.	22.	40.	26.	101	1126
19.	11.	44.	33.	14.	25.	13.	77	1127
46.	13.	58.	47.	44.	47.	25.	105	1128
32.	18.	56.	47.	32.	44.	28.	103	1129
22.	19.	54.	41.	22.	30.	18.	95	1130
27.	24.	56.	45.	22.	35.	30.	101	1131
35.	19.	55.	47.	19.	39.	20.	102	1132
26.	17.	46.	46.	24.	30.	18.	92	1133
29.	17.	54.	47.	25.	35.	26.	101	1134
33.	15.	53.	45.	21.	34.	21.	98	1135
34.	21.	56.	43.	33.	42.	19.	99	1136

AC	PS	V	Q	SMU	AC	PS	TOT	ID
14.	7.	33.	25.	13.	15.	11.	58	1137
20.	6.	53.	27.	12.	15.	9.	80	1138
19.	11.	45.	35.	18.	20.	16.	80	1139
16.	12.	54.	42.	13.	26.	13.	96	1140
39.	8.	57.	49.	29.	36.	16.	106	1141
17.	10.	44.	31.	8.	15.	15.	75	1142
21.	13.	50.	36.	18.	34.	9.	86	1143
15.	7.	55.	35.	13.	27.	12.	90	1144
28.	14.	57.	47.	26.	36.	17.	104	1145
15.	4.	45.	32.	13.	24.	7.	77	1146
8.	12.	45.	33.	16.	16.	12.	78	1147
14.	8.	33.	44.	8.	24.	9.	77	1148
12.	11.	43.	31.	15.	26.	16.	74	1149
14.	10.	42.	41.	12.	22.	9.	83	1150
21.	7.	54.	42.	18.	36.	13.	96	1151
18.	12.	53.	48.	20.	30.	16.	101	1152
22.	15.	50.	43.	16.	33.	17.	93	1153
14.	10.	46.	24.	15.	19.	11.	70	1154
25.	14.	56.	45.	27.	31.	19.	101	1155
11.	11.	51.	40.	12.	27.	17.	91	1156
27.	20.	53.	45.	15.	37.	18.	98	1157
17.	11.	39.	32.	16.	16.	13.	71	1158
13.	14.	46.	33.	12.	15.	8.	79	1159
20.	13.	54.	48.	24.	30.	19.	102	1160
10.	6.	53.	26.	4.	10.	8.	79	1161
23.	15.	54.	45.	18.	33.	23.	99	1162
33.	17.	55.	44.	19.	33.	14.	99	1163
17.	14.	39.	33.	17.	25.	16.	72	1164
16.	11.	52.	45.	15.	21.	9.	97	1165
31.	14.	53.	44.	14.	27.	11.	97	1166
13.	12.	52.	39.	23.	27.	21.	91	1167
40.	20.	58.	47.	38.	44.	26.	105	1168
35.	14.	59.	43.	31.	36.	19.	102	1169
18.	16.	55.	47.	22.	27.	19.	102	1170

AC	PS	V	Q	SMU	AC	PS	TOT	ID
29.	12.	51.	37.	24.	24.	21.	88	1171
20.	12.	31.	34.	21.	22.	21.	65	1172
16.	17.	55.	43.	18.	34.	22.	98	1173
20.	10.	46.	37.	19.	20.	21.	83	1174
42.	18.	55.	46.	29.	43.	20.	101	1175
20.	7.	49.	42.	16.	25.	9.	91	1176
41.	17.	56.	45.	38.	42.	27.	101	1177
10.	9.	35.	31.	17.	23.	12.	66	1178
19.	6.	40.	33.	16.	18.	15.	73	1179
33.	10.	56.	42.	24.	32.	14.	98	1180
11.	8.	47.	26.	5.	10.	11.	73	1181
27.	19.	57.	42.	22.	34.	21.	99	1182
24.	13.	54.	41.	22.	33.	19.	95	1183
23.	12.	51.	37.	19.	33.	14.	88	1184
19.	10.	48.	38.	17.	25.	20.	86	1185
9.	9.	23.	25.	10.	9.	9.	48	1186
26.	10.	56.	37.	15.	29.	15.	93	1187
20.	9.	48.	36.	22.	24.	13.	84	1188
13.	10.	46.	30.	13.	12.	12.	76	1189
16.	5.	42.	31.	9.	14.	14.	73	1190
21.	11.	52.	40.	14.	27.	10.	92	1191
17.	14.	41.	32.	20.	31.	18.	73	1192
13.	12.	42.	30.	10.	14.	9.	72	1193
9.	5.	42.	18.	6.	12.	5.	60	1194
16.	11.	42.	29.	14.	15.	21.	71	1195
10.	6.	38.	30.	13.	10.	6.	68	1196
19.	12.	51.	40.	21.	33.	15.	91	1197
32.	21.	57.	46.	41.	44.	23.	103	1198
16.	12.	45.	35.	16.	28.	20.	80	1199
30.	19.	49.	42.	18.	36.	16.	91	1200
20.	14.	46.	40.	19.	26.	14.	86	1201
17.	7.	47.	39.	20.	30.	17.	86	1202
11.	7.	37.	26.	3.	20.	5.	63	1203
10.	8.	32.	34.	16.	26.	9.	66	1204

AC	PS	V	Q	SMU	AC	PS	TOT	ID
24.	17.	51.	45.	23.	33.	23.	96	1205
15.	7.	40.	26.	13.	17.	9.	66	1206
16.	15.	55.	40.	18.	26.	19.	95	1207
23.	18.	54.	40.	27.	31.	17.	94	1208
22.	10.	37.	40.	14.	26.	17.	77	1209
16.	10.	38.	25.	8.	23.	8.	63	1210
19.	15.	44.	36.	19.	23.	15.	80	1211
27.	19.	57.	47.	24.	40.	19.	104	1212
23.	11.	47.	33.	21.	28.	16.	80	1213
16.	12.	38.	29.	15.	27.	14.	67	1214
23.	13.	51.	39.	22.	36.	17.	90	1215
26.	18.	43.	35.	18.	35.	17.	78	1216
21.	10.	53.	31.	16.	36.	16.	84	1217
21.	10.	49.	34.	18.	26.	12.	83	1218
16.	11.	52.	32.	18.	30.	16.	84	1219
20.	17.	45.	31.	15.	20.	17.	76	1220
25.	13.	49.	44.	16.	29.	21.	93	1221
13.	5.	50.	29.	6.	17.	9.	79	1222
17.	11.	53.	38.	20.	34.	11.	91	1223
22.	10.	56.	35.	22.	22.	13.	91	1224
17.	6.	48.	27.	17.	27.	12.	75	1225
20.	11.	58.	37.	15.	23.	11.	95	1226
35.	18.	58.	45.	26.	39.	20.	103	1227
30.	19.	56.	44.	27.	39.	16.	100	1228
15.	8.	38.	37.	17.	18.	14.	75	1229
38.	14.	58.	47.	30.	39.	18.	105	1230
18.	11.	50.	29.	20.	21.	11.	79	1231
23.	6.	45.	39.	9.	24.	10.	84	1232
23.	10.	56.	35.	13.	28.	11.	91	1233
34.	22.	56.	48.	24.	34.	21.	104	1234
15.	6.	45.	35.	7.	27.	12.	80	1235
13.	15.	45.	38.	17.	29.	13.	83	1236
17.	10.	40.	25.	10.	18.	11.	65	1237
26.	18.	54.	49.	20.	39.	14.	103	1238

AC	PS	V	Q	SMU	AC	PS	TOT	ID
29.	14.	47.	49.	22.	39.	15.	96	1239
29.	12.	56.	48.	15.	35.	17.	104	1240
30.	17.	48.	40.	12.	29.	18.	88	1241
12.	8.	48.	30.	8.	18.	7.	78	1242
17.	11.	56.	37.	17.	33.	12.	93	1243
42.	19.	54.	47.	32.	43.	17.	101	1244
18.	13.	54.	40.	22.	31.	17.	94	1245
16.	6.	34.	42.	12.	20.	9.	76	1246
23.	10.	49.	44.	17.	31.	9.	93	1247
20.	15.	42.	42.	13.	22.	15.	84	1248
21.	14.	50.	38.	15.	35.	8.	88	1249
19.	14.	56.	43.	22.	34.	19.	99	1250
30.	11.	50.	44.	17.	33.	16.	94	1251
40.	11.	48.	48.	32.	39.	19.	96	1252
15.	6.	48.	24.	7.	14.	8.	72	1253
19.	11.	51.	38.	15.	26.	17.	89	1254
14.	9.	49.	22.	14.	17.	13.	71	1255
17.	11.	55.	33.	11.	22.	10.	88	1256
20.	13.	49.	38.	19.	29.	13.	87	1257
17.	11.	59.	48.	27.	39.	19.	107	1258
36.	20.	54.	48.	29.	40.	23.	102	1259
18.	6.	31.	30.	9.	25.	13.	61	1260
18.	12.	52.	37.	12.	25.	15.	89	1261
21.	13.	38.	33.	22.	25.	12.	71	1262
15.	15.	46.	33.	17.	15.	14.	79	1263
24.	12.	44.	40.	22.	26.	19.	84	1264
17.	15.	26.	33.	10.	20.	11.	59	1265
19.	14.	42.	32.	17.	28.	15.	74	1266
20.	13.	41.	37.	18.	27.	20.	78	1267
25.	12.	51.	33.	16.	35.	16.	84	1268
34.	19.	53.	46.	23.	38.	25.	99	1269
24.	11.	49.	31.	11.	23.	17.	80	1270
21.	25.	37.	36.	12.	25.	15.	73	1271
9.	8.	34.	25.	10.	11.	10.	59	1272

AC	PS	V	Q	SMU	AC	PS	TOT	ID
16.	9.	41.	18.	9.	13.	10.	59	1273
9.	10.	23.	22.	17.	16.	12.	45	1274
15.	7.	46.	23.	9.	13.	13.	69	1275
31.	16.	54.	40.	24.	30.	21.	94	1276
28.	20.	47.	45.	19.	33.	26.	92	1277
30.	15.	50.	42.	11.	30.	14.	92	1278
23.	12.	42.	40.	24.	25.	16.	82	1279
18.	16.	41.	38.	15.	33.	14.	79	1280
22.	12.	38.	39.	17.	27.	17.	77	1281
26.	11.	48.	38.	16.	37.	19.	86	1282
13.	6.	41.	25.	14.	22.	14.	66	1283
26.	17.	42.	32.	18.	35.	19.	74	1284
20.	12.	40.	36.	20.	28.	19.	76	1285
16.	11.	28.	25.	17.	19.	13.	53	1286
13.	9.	27.	25.	14.	16.	9.	52	1287
20.	17.	47.	29.	16.	17.	11.	76	1288
16.	13.	36.	26.	18.	19.	15.	62	1289
25.	9.	48.	42.	27.	30.	16.	90	1290
20.	9.	49.	32.	18.	28.	12.	81	1291
26.	12.	55.	42.	17.	32.	15.	97	1292
26.	17.	54.	43.	14.	33.	13.	97	1293
16.	11.	36.	34.	11.	14.	8.	70	1294
12.	13.	25.	31.	20.	14.	14.	56	1295
20.	12.	45.	36.	14.	19.	15.	81	1296
11.	12.	34.	22.	8.	11.	10.	56	1297
25.	16.	50.	37.	25.	30.	12.	87	1298
18.	17.	35.	29.	11.	23.	17.	64	1299
18.	7.	35.	27.	9.	17.	16.	62	1300
13.	4.	42.	24.	15.	12.	10.	66	1301
37.	15.	55.	47.	27.	35.	19.	102	1302
15.	10.	42.	28.	6.	19.	8.	70	1303
23.	18.	38.	33.	14.	28.	11.	71	1304
23.	19.	37.	41.	20.	27.	16.	78	1305
28.	16.	54.	44.	18.	34.	20.	98	1306

AC	PS	V	Q	SMU	AC	PS	TOT	ID
17.	11.	44.	39.	16.	22.	14.	83	1307
18.	10.	50.	34.	24.	24.	14.	84	1308
24.	11.	31.	27.	17.	27.	7.	58	1309
26.	11.	45.	40.	15.	25.	17.	85	1310
20.	15.	47.	25.	15.	25.	16.	72	1311
25.	18.	42.	37.	16.	21.	17.	79	1312

Beginning on the following page, the scores obtained by the Exploring Modern Mathematics students are listed.

AC	PS	V	Q	SMU	AC	PS	TOT	ID
36.	19.	47.	48.	31.	36.	25.	95	2001
29.	11.	54.	31.	12.	32.	9.	85	2002
14.	13.	38.	30.	12.	23.	13.	68	2003
21.	9.	54.	33.	16.	31.	9.	87	2004
25.	18.	41.	39.	15.	31.	21.	80	2005
22.	17.	50.	33.	17.	20.	13.	83	2006
22.	18.	56.	38.	19.	25.	10.	94	2007
24.	14.	40.	37.	21.	28.	15.	77	2008
24.	8.	58.	33.	18.	27.	17.	91	2009
29.	17.	51.	45.	32.	36.	18.	96	2010
29.	19.	53.	46.	24.	42.	21.	99	2011
20.	19.	51.	32.	15.	23.	13.	83	2012
12.	8.	43.	39.	20.	25.	10.	82	2013
30.	14.	57.	47.	24.	37.	13.	104	2014
15.	11.	48.	42.	19.	25.	12.	90	2015
15.	11.	56.	36.	14.	22.	8.	92	2016
24.	13.	41.	39.	22.	36.	12.	80	2017
33.	16.	54.	47.	32.	45.	21.	101	2018
24.	15.	52.	44.	22.	33.	16.	96	2019
22.	10.	57.	32.	26.	37.	20.	89	2020
27.	14.	51.	44.	25.	36.	17.	95	2021
29.	14.	52.	42.	21.	40.	15.	94	2022
23.	23.	51.	38.	17.	23.	23.	89	2023
35.	20.	55.	44.	33.	41.	27.	99	2024
32.	11.	53.	39.	6.	26.	14.	92	2025
32.	10.	53.	46.	26.	33.	22.	99	2026
12.	11.	53.	33.	16.	21.	12.	86	2027
24.	13.	50.	36.	18.	33.	13.	86	2028
30.	17.	56.	41.	24.	39.	20.	97	2029
23.	10.	55.	36.	20.	22.	17.	91	2030
24.	15.	51.	36.	27.	35.	20.	87	2031
12.	12.	34.	27.	17.	10.	9.	61	2032
16.	15.	39.	39.	26.	30.	21.	78	2033
38.	20.	53.	46.	29.	40.	20.	99	2034

AC	PS	V	Q	SMU	AC	PS	TOT	ID
29.	11.	49.	35.	22.	32.	12.	84	2035
24.	19.	54.	42.	19.	34.	21.	96	2036
33.	16.	49.	43.	32.	38.	23.	92	2037
21.	14.	43.	39.	25.	24.	23.	82	2038
20.	13.	44.	37.	20.	25.	12.	81	2039
11.	12.	44.	30.	9.	16.	13.	74	2040
27.	16.	39.	39.	23.	37.	19.	78	2041
23.	15.	45.	41.	29.	38.	17.	86	2042
21.	11.	41.	27.	23.	32.	18.	68	2043
26.	14.	50.	37.	22.	26.	22.	87	2044
22.	9.	45.	35.	21.	27.	17.	80	2045
24.	15.	55.	42.	26.	35.	18.	97	2046
39.	21.	56.	42.	30.	43.	24.	98	2047
20.	17.	48.	39.	11.	20.	18.	87	2048
34.	18.	55.	41.	24.	41.	26.	96	2049
32.	18.	43.	40.	24.	38.	21.	83	2050
31.	16.	50.	41.	30.	35.	17.	91	2051
23.	16.	41.	39.	22.	32.	16.	80	2052
20.	17.	43.	35.	22.	26.	18.	78	2053
31.	11.	51.	39.	24.	34.	18.	90	2054
27.	15.	53.	37.	22.	37.	21.	90	2055
45.	17.	59.	47.	40.	45.	25.	106	2056
32.	20.	48.	46.	26.	35.	17.	94	2057
16.	16.	46.	35.	14.	25.	14.	81	2058
21.	17.	42.	32.	27.	33.	16.	74	2059
41.	28.	60.	49.	43.	44.	27.	109	2060
19.	16.	46.	32.	22.	20.	16.	78	2061
16.	17.	44.	44.	21.	31.	21.	88	2062
14.	11.	49.	32.	20.	24.	13.	81	2063
25.	12.	39.	34.	17.	20.	16.	73	2064
23.	12.	30.	18.	28.	29.	16.	48	2065
28.	13.	57.	44.	27.	41.	21.	101	2066
32.	12.	55.	37.	29.	39.	19.	92	2067
13.	11.	39.	38.	24.	16.	10.	77	2068

AC	PS	V	Q	SMU	AC	PS	TOT	ID
39.	18.	56.	47.	35.	41.	15.	103	2069
22.	15.	56.	39.	21.	28.	19.	95	2070
16.	11.	38.	35.	11.	26.	11.	73	2071
15.	7.	38.	24.	18.	12.	11.	62	2072
19.	14.	45.	33.	14.	19.	13.	78	2073
26.	14.	50.	40.	29.	37.	18.	90	2074
29.	18.	51.	47.	19.	33.	27.	98	2075
20.	16.	44.	36.	19.	25.	16.	80	2076
11.	10.	25.	24.	9.	17.	13.	49	2077
27.	15.	39.	40.	19.	30.	22.	79	2078
19.	15.	55.	38.	19.	27.	22.	93	2079
27.	16.	52.	40.	26.	28.	27.	92	2080
12.	12.	48.	27.	11.	20.	12.	75	2081
35.	18.	53.	44.	31.	40.	22.	97	2082
24.	20.	42.	40.	24.	28.	19.	82	2083
24.	20.	39.	39.	25.	31.	14.	78	2084
22.	14.	36.	39.	17.	28.	19.	75	2085
24.	22.	51.	45.	30.	34.	23.	96	2086
17.	14.	51.	38.	25.	26.	20.	89	2087
16.	11.	47.	30.	17.	21.	13.	77	2088
28.	16.	52.	46.	23.	35.	17.	98	2089
20.	16.	46.	48.	19.	30.	17.	94	2090
31.	14.	57.	46.	35.	39.	16.	103	2091
29.	15.	47.	37.	25.	31.	23.	84	2092
32.	14.	38.	39.	31.	35.	14.	77	2093
26.	15.	50.	43.	29.	32.	18.	93	2094
23.	14.	46.	37.	25.	35.	18.	83	2095
35.	22.	58.	48.	35.	43.	27.	106	2096
13.	8.	49.	37.	18.	23.	17.	86	2097
21.	14.	49.	45.	18.	33.	15.	94	2098
25.	13.	42.	39.	11.	22.	9.	81	2099
39.	20.	55.	49.	24.	41.	22.	104	2100
16.	11.	36.	38.	7.	27.	11.	74	2101
16.	12.	42.	36.	12.	23.	10.	78	2102

AC	PS	V	Q	SMU	AC	PS	TOT	ID
19.	18.	32.	41.	17.	29.	17.	73	2103
13.	12.	42.	18.	11.	15.	6.	60	2104
28.	19.	48.	42.	16.	34.	21.	90	2105
13.	14.	49.	36.	13.	22.	13.	85	2106
23.	13.	46.	39.	21.	25.	18.	85	2107
19.	11.	30.	34.	15.	27.	12.	64	2108
11.	15.	39.	32.	10.	18.	16.	71	2109
19.	9.	34.	34.	12.	25.	11.	68	2110
17.	17.	24.	41.	13.	18.	10.	65	2111
13.	8.	44.	33.	13.	23.	8.	77	2112
22.	9.	48.	40.	18.	30.	14.	88	2113
15.	12.	39.	41.	13.	26.	23.	80	2114
21.	15.	52.	46.	15.	35.	17.	98	2115
13.	11.	32.	39.	15.	21.	14.	71	2116
17.	14.	45.	35.	15.	24.	12.	80	2117
7.	5.	27.	18.	13.	7.	11.	45	2118
42.	25.	50.	49.	38.	47.	26.	99	2119
25.	10.	47.	41.	26.	29.	13.	88	2120
31.	20.	55.	47.	32.	40.	28.	102	2121
26.	20.	48.	38.	21.	32.	15.	86	2122
30.	17.	45.	45.	21.	27.	18.	90	2123
15.	12.	39.	37.	10.	24.	14.	76	2124
30.	13.	53.	41.	27.	38.	18.	94	2125
15.	11.	33.	34.	19.	15.	13.	67	2126
26.	8.	48.	42.	21.	28.	10.	90	2127
29.	15.	52.	42.	16.	29.	19.	94	2128
26.	20.	51.	44.	28.	37.	19.	95	2129
41.	17.	51.	47.	31.	42.	22.	98	2130
24.	15.	46.	44.	19.	31.	18.	90	2131
31.	15.	54.	47.	24.	34.	15.	101	2132
29.	15.	55.	41.	19.	25.	18.	96	2133
41.	15.	58.	45.	38.	42.	17.	103	2134
35.	25.	56.	48.	29.	42.	20.	104	2135
24.	12.	39.	35.	26.	29.	19.	74	2136

AC	PS	V	Q	SMU	AC	PS	TOT	ID
19.	12.	48.	38.	17.	26.	8.	86	2137
12.	13.	51.	24.	10.	15.	6.	75	2138
31.	15.	52.	46.	29.	39.	18.	98	2139
21.	8.	53.	32.	21.	29.	14.	85	2140
38.	17.	54.	44.	28.	44.	17.	98	2141
30.	14.	58.	40.	24.	38.	21.	98	2142
22.	19.	43.	41.	17.	34.	21.	84	2143
34.	21.	54.	46.	24.	35.	20.	100	2144

Beginning on the following page, the scores obtained by the *Winston Mathematics* students are listed.

AC	PS	V	Q	SMU	AC	PS	TOT	ID
33.	12.	53.	42.	13.	36.	13.	95	3001
38.	14.	52.	48.	18.	42.	20.	100	3002
26.	16.	43.	37.	9.	34.	16.	80	3003
29.	13.	50.	43.	12.	37.	15.	93	3004
16.	11.	32.	34.	10.	21.	12.	66	3005
30.	14.	56.	38.	12.	30.	15.	94	3006
36.	21.	51.	45.	15.	37.	20.	96	3007
25.	9.	52.	44.	18.	36.	20.	96	3008
25.	14.	55.	36.	18.	26.	12.	91	3009
42.	16.	57.	48.	26.	44.	19.	105	3010
32.	18.	57.	45.	17.	38.	25.	102	3011
35.	17.	57.	45.	26.	38.	23.	102	3012
19.	15.	45.	33.	15.	26.	20.	78	3013
37.	20.	57.	47.	22.	43.	23.	104	3014
42.	21.	55.	50.	31.	43.	28.	105	3015
29.	11.	54.	37.	14.	28.	19.	91	3016
29.	14.	50.	42.	5.	25.	15.	92	3017
36.	12.	51.	48.	15.	38.	20.	99	3018
36.	21.	53.	35.	20.	26.	17.	88	3019
31.	19.	49.	41.	14.	27.	20.	90	3020
32.	19.	52.	45.	16.	38.	19.	97	3021
31.	13.	53.	48.	16.	41.	21.	101	3022
26.	9.	49.	43.	16.	32.	12.	92	3023
14.	12.	40.	37.	18.	19.	16.	77	3024
41.	24.	56.	49.	29.	45.	28.	105	3025
38.	24.	53.	47.	18.	44.	25.	100	3026
24.	7.	56.	38.	16.	31.	13.	94	3027
24.	17.	54.	39.	6.	25.	15.	93	3028
21.	11.	52.	37.	15.	27.	11.	89	3029
18.	9.	46.	17.	6.	23.	12.	63	3030
32.	21.	46.	48.	17.	40.	24.	94	3031
28.	13.	41.	48.	16.	41.	25.	89	3032
33.	15.	46.	47.	18.	39.	22.	93	3033
27.	15.	40.	40.	11.	37.	23.	80	3034

AC	PS	V	Q	SMU	AC	PS	TOT	ID
18.	11.	51.	33.	12.	27.	16.	84	3035
33.	10.	51.	45.	15.	35.	17.	96	3036
26.	4.	37.	44.	10.	31.	20.	81	3037
27.	15.	39.	40.	4.	28.	16.	79	3038
19.	7.	47.	29.	10.	25.	11.	76	3039
28.	9.	51.	40.	8.	34.	16.	91	3040
21.	13.	44.	34.	10.	27.	16.	78	3041
17.	17.	37.	40.	7.	21.	15.	77	3042
27.	17.	55.	38.	8.	32.	22.	93	3043
22.	14.	41.	44.	7.	34.	17.	85	3044
20.	6.	47.	34.	16.	30.	18.	81	3045
21.	14.	50.	32.	9.	23.	18.	82	3046
32.	14.	47.	43.	13.	41.	21.	90	3047
33.	19.	48.	46.	11.	39.	18.	94	3048
16.	8.	34.	34.	10.	20.	11.	68	3049
37.	16.	46.	39.	22.	34.	20.	85	3050
23.	13.	44.	46.	7.	36.	19.	90	3051
25.	19.	54.	45.	13.	33.	18.	99	3052
22.	14.	36.	40.	9.	31.	15.	76	3053
24.	15.	44.	34.	14.	25.	17.	78	3054
20.	9.	39.	41.	14.	26.	25.	80	3055
22.	18.	50.	39.	15.	32.	21.	89	3056
40.	25.	53.	49.	17.	41.	31.	102	3057
29.	22.	50.	42.	18.	31.	15.	92	3058
21.	10.	32.	39.	11.	28.	15.	71	3059
14.	8.	28.	29.	9.	20.	8.	57	3060
18.	13.	37.	33.	10.	24.	10.	70	3061
29.	21.	57.	48.	22.	41.	22.	105	3062
21.	20.	48.	41.	19.	35.	16.	89	3063
25.	13.	43.	44.	16.	37.	17.	87	3064
37.	14.	54.	40.	22.	41.	21.	94	3065
28.	9.	53.	44.	19.	30.	19.	97	3066
27.	14.	46.	47.	18.	34.	24.	93	3067
19.	11.	39.	37.	15.	32.	9.	76	3068

AC	PS	V	Q	SMU	AC	PS	TOT	ID
26.	17.	56.	36.	24.	31.	17.	92	3069
27.	10.	39.	40.	18.	27.	22.	79	3070
21.	13.	38.	44.	19.	23.	19.	82	3071
34.	18.	54.	43.	19.	38.	25.	97	3072
20.	12.	45.	41.	15.	33.	17.	86	3073
38.	27.	59.	47.	24.	42.	23.	106	3074
36.	17.	57.	43.	19.	35.	16.	100	3075
20.	7.	48.	22.	9.	21.	8.	70	3076
31.	15.	50.	46.	21.	36.	18.	96	3077
29.	10.	42.	32.	18.	21.	15.	74	3078
17.	9.	40.	36.	14.	22.	14.	76	3079
29.	15.	50.	42.	13.	30.	14.	92	3080
31.	19.	52.	43.	21.	34.	19.	95	3081
37.	14.	54.	40.	17.	38.	18.	94	3082
32.	18.	47.	35.	21.	37.	25.	82	3083
25.	19.	50.	38.	15.	33.	25.	88	3084
34.	16.	53.	48.	24.	44.	20.	101	3085
13.	9.	34.	16.	12.	19.	12.	50	3086
22.	7.	32.	36.	12.	21.	17.	68	3087
33.	19.	52.	45.	23.	40.	19.	97	3088
22.	12.	51.	25.	12.	21.	12.	76	3089
26.	9.	33.	32.	14.	26.	13.	65	3090
23.	7.	56.	41.	13.	28.	19.	97	3091
32.	19.	57.	50.	33.	45.	24.	107	3092
28.	20.	49.	44.	20.	31.	23.	93	3093
35.	14.	57.	48.	17.	40.	15.	105	3094
24.	13.	54.	41.	14.	27.	18.	95	3095
39.	17.	56.	46.	24.	42.	20.	102	3096
47.	23.	57.	49.	35.	46.	27.	106	3097
20.	13.	41.	40.	15.	35.	20.	81	3098
18.	7.	46.	34.	19.	27.	6.	80	3099
33.	16.	49.	46.	24.	38.	24.	95	3100
41.	14.	50.	47.	22.	43.	23.	97	3101
35.	17.	55.	43.	19.	39.	24.	98	3102

AC	PS	V	Q	SMU	AC	PS	TOT	ID
40.	25.	54.	49.	28.	43.	30.	103	3103
24.	11.	50.	47.	17.	26.	13.	97	3104
35.	12.	47.	44.	21.	31.	16.	91	3105
30.	19.	52.	46.	20.	35.	27.	98	3106
33.	18.	50.	44.	24.	38.	21.	94	3107
32.	13.	46.	44.	16.	38.	19.	90	3108
16.	13.	49.	28.	11.	23.	16.	77	3109
36.	13.	55.	43.	29.	39.	17.	98	3110
32.	17.	48.	43.	15.	37.	25.	91	3111
41.	21.	53.	47.	23.	44.	29.	100	3112
31.	9.	51.	39.	13.	36.	8.	90	3113
42.	20.	50.	45.	26.	45.	30.	95	3114
28.	16.	51.	47.	19.	35.	16.	98	3115
21.	5.	53.	35.	12.	29.	11.	88	3116
46.	24.	58.	47.	35.	45.	28.	105	3117
42.	30.	56.	50.	33.	47.	32.	106	3118
34.	13.	48.	48.	16.	36.	18.	96	3119
36.	17.	55.	46.	27.	41.	22.	101	3120
33.	15.	50.	47.	19.	39.	17.	97	3121
37.	20.	53.	48.	17.	40.	25.	101	3122
26.	15.	51.	42.	14.	37.	19.	93	3123
32.	20.	53.	46.	18.	34.	26.	99	3124
40.	17.	57.	49.	18.	39.	25.	106	3125
38.	13.	51.	47.	18.	39.	21.	98	3126
28.	10.	46.	35.	8.	26.	15.	81	3127
29.	12.	53.	43.	15.	38.	23.	96	3128
20.	16.	52.	38.	11.	26.	18.	90	3129
11.	10.	41.	25.	6.	11.	8.	66	3130
20.	15.	44.	37.	6.	24.	18.	81	3131
17.	12.	45.	38.	16.	23.	11.	83	3132
12.	8.	40.	35.	7.	21.	15.	75	3133
35.	14.	54.	45.	22.	39.	19.	99	3134
28.	23.	48.	48.	14.	34.	18.	96	3135
15.	5.	44.	19.	6.	16.	9.	63	3136

AC	PS	V	Q	SMU	AC	PS	TOT	ID
33.	22.	54.	47.	28.	37.	25.	101	3137
13.	3.	42.	27.	12.	20.	9.	69	3138
34.	15.	53.	44.	25.	38.	20.	97	3139
31.	19.	52.	50.	12.	34.	21.	102	3140
16.	10.	35.	26.	9.	19.	11.	61	3141
31.	21.	44.	46.	15.	38.	24.	90	3142
37.	23.	55.	49.	22.	40.	27.	104	3143
19.	7.	50.	2 .	15.	21.	12.	79	3144
37.	24.	57.	50.	21.	41.	32.	107	3145
23.	16.	59.	40.	17.	36.	15.	99	3146
17.	8.	48.	33.	10.	27.	15.	81	3147
17.	7.	46.	31.	10.	24.	10.	77	3148

SUMMARY OF ANALYSES OF VARIANCE OF SEPTEMBER TEST SCORES

ANALYSIS OF VARIANCE: SEPTEMBER AC SCORES

SOURCE	DF	SS	MS	F
TOT	603	.41659840E+05		
GRP	2	.18978600E+04	.94893000E+03	.14343021E+02
WTH	601	.39761980E+05	.66159700E+02	

ANALYSIS OF VARIANCE: SEPTEMBER PS SCORES

SOURCE	DF	SS	MS	F
TOT	603	.13319550E+05		
GRP	2	.99210000E+02	.49605000E+02	.22550559E+01
WTH	601	.13220340E+05	.21997237E+02	

ANALYSIS OF VARIANCE: SEPTEMBER V SCORES

SOURCE	DF	SS	MS	F
TOT	603	.32257800E+05		
GRP	2	.12880000E+03	.64400000E+02	.12046562E+01
WTH	601	.32129000E+05	.53459234E+02	

ANALYSIS OF VARIANCE: SEPTEMBER Q SCORES

SOURCE	DF	SS	MS	F
TOT	603	.30181170E+05		
GRP	2	.44276000E+03	.22138000E+03	.44739910E+01
WTH	601	.29738410E+05	.49481547E+02	

CHI SQUARE TEST ON CELL FREQUENCIES

	HI	MED	LO	
STM	102 (100)	98 (105)	112 (107)	312
EMM	36 (46)	56 (48)	52 (50)	144
CON	55 (47)	49 (50)	44 (51)	148
	193	203	208	604

Numbers in parentheses are expected cell frequencies.

$O - E$	$(O - E)^2$	$\frac{(O - E)^2}{E}$
2	4	.04
-7	49	.47
5	25	.23
-10	100	2.12
7	49	1.02
2	4	.08
8	64	1.36
-1	1	.02
-7	49	.96

$$\chi^2 = 6.30$$

df = 4

$$\chi^2_{.10}(4df) = 7.8$$

$$\chi^2_{.01}(4df) = 13.3$$

COMPUTATIONAL FORMULAS FOR TWO-WAY UNWEIGHTED MEANS ANALYSES OF VARIANCE AND ANALYSIS OF COVARIANCE

Definition of Symbols

	b_1	b_2	b_3	
a_1	n_{11}	n_{12}	n_{13}	STM
a_2	n_{21}	n_{22}	n_{23}	EMM
a_3	n_{31}	n_{32}	n_{33}	CON
	HI	HAV	AV	

Harmonic mean of cell frequencies, $n_h = \dfrac{pq}{\sum_{j=1}^{p} \sum_{k=1}^{q} \dfrac{1}{n_{jk}}}$

$p = 3 \quad q = 3$

$\overline{AB}_{x_{jk}}$ is the covariate mean for cell jk.

$\overline{AB}_{y_{jk}}$ is the criterion mean for cell jk.

$$G_x = \sum_j \sum_k \overline{AB}_{x_{jk}} \qquad G_y = \sum_j \sum_k \overline{AB}_{y_{jk}}$$

$$A_{x_j} = \sum_k \overline{AB}_{x_{jk}} \qquad B_{x_k} = \sum_j \overline{AB}_{x_{jk}}$$

Programs: $A_{xx} = SS_{Ax}$ $A_{yy} = SS_{Ay}$

Ability Levels: $B_{xx} = SS_{Bx}$ $B_{yy} = SS_{Ay}$

Interaction: $AB_{xx} = SS_{ABx}$ $AB_{yy} = SS_{ABy}$

Error: $E_{xx} = SS_{WTHx}$ $E_{yy} = SS_{WTHy}$

A'_{yy} , B'_{yy} , AB'_{yy} , and E'_{yy} represent the sums of squares for the criterion adjusted for the effect of the covariate.

Computational Formulas

(Adaptation from Winer, pp. 241-244 and p. 605.)

Covariate Variance Terms	Covariance Terms	Criterion Variance Terms
$(1x) = \frac{G_x^2}{pq}$	$(1xy) = \frac{G_x G_y}{pq}$	$(1y) = \frac{G_y^2}{pq}$
$(2x) = \sum X^2$	$(2xy) = \sum XY$	$(2y) = \sum Y^2$
$(3x) = \frac{\sum_j A_{x_j}^2}{q}$	$(3xy) = \frac{\sum_j A_{x_j} A_{y_j}}{q}$	$(3y) = \frac{\sum_j A_{y_j}^2}{q}$
$(4x) = \frac{\sum_k B_{x_k}^2}{p}$	$(4xy) = \frac{\sum_k B_{x_k} B_{y_k}}{p}$	$(4y) = \frac{\sum_k B_{y_k}^2}{p}$
$(5x) = \sum_j \sum_k (\overline{AB}_{x_{jk}})^2$	$(5xy) = \sum_j \sum_k \overline{AB}_{x_{jk}} \overline{AB}_{y_{jk}}$	$(5y) = \sum_j \sum_k (\overline{AB}_{y_{jk}})^2$

ANOVA Formulas

Covariate	Criterion
$A_{xx} = n_h[(3x) - (1x)]$	$A_{yy} = n_h[(3y) - (1y)]$
$B_{xx} = n_h[(4x) - (1x)]$	$B_{yy} = n_h[(4y) - (1y)]$
$AB_{xx} = n_h[(5x)-(3x)-(4x)+(1x)]$	$AB_{yy} = n_h[(5y)-(3y)-(4y)+(1y)]$
$E_{xx} = \sum_j \sum_k [\sum_i X_{ijk}^2 - n_{jk}\overline{AB}_{x_{jk}}^2]$	$E_{yy} = \sum_j \sum_k [\sum_i Y_{ijk}^2 - n_{jk}\overline{AB}_{y_{jk}}^2]$

ANCOVA Formulas

$$A_{xy} = n_h[(3xy) - (1xy)]$$

$$B_{xy} = n_h[(4xy) - (1xy)]$$

$$AB_{xy} = n_h[(5xy) - (3xy) - (4xy) + (1xy)]$$

$$E_{xy} = \sum_j \sum_k E_{xy_{jk}}$$

$$= \sum_j \sum_k [\ \sum_i XY_{ijk} - n_{jk}\overline{AB}_{x_{jk}}\overline{AB}_{y_{jk}}]$$

$$E'_{yy} = E_{yy} - \frac{E^2_{xy}}{E_{xx}}$$

$$A'_{yy} = A_{yy} - \frac{(A_{xy} + E_{xy})^2}{A_{xx} + E_{xx}} + \frac{E^2_{xy}}{E_{xx}}$$

$$B'_{yy} = B_{yy} - \frac{(B_{xy} + E_{xy})^2}{B_{xx} + E_{xx}} + \frac{E^2_{xy}}{E_{xx}}$$

$$AB'_{yy} = AB_{yy} - \frac{(AB_{xy} + E_{xy})^2}{AB_{xx} + E_{xx}} + \frac{E^2_{xy}}{E_{xx}}$$

SUMMARY OF ANALYSES OF VARIANCE ON COVARIATE AND CRITERION FOR ANALYSIS OF COVARIANCE ON MARCH AC SCORES

ANALYSIS OF VARIANCE: SEPTEMBER AC SCORES

SOURCE	SS	df	MS	F
A	A_{xx} = 1,104.01	2	552.01	17.50
B	B_{xx} =18,473.21	2	9,236.60	292.10
AB	AB_{xx} = 116.27	4	29.07	0.92
WITHIN	E_{xx} =18,787.97	595	31.58	

ANALYSIS OF VARIANCE: MARCH AC SCORES

SOURCE	SS	df	MS	F
A	A_{yy} = 637.11	2	318.55	10.52
B	B_{yy} = 19,689.96	2	9,844.98	279.16
AB	AB_{yy} = 166.06	4	41.52	1.37
WITHIN	E_{yy} = 18,019.64	595	30.29	

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